MASTER'S THESIS

A THESIS SUBMITTED IN FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER IN CYBERNETICS AND ROBOTICS AT CZECH TECHNICAL UNIVERSITY (CVUT) AND FOR THE DEGREE OF JOINT MASTER PROGRAM IN SPACE SCIENCE AND TECHNOLOGY - SPACEMASTER (LTU).

STUDY OF LAUNCHER RECOVERY SYSTEMS

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MASTER'S THESIS ASSIGNMENT

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II. Master's thesis details

Master's thesis title in English:

Study of launcher recovery systems

Master's thesis title in Czech:

Studie systémů obnovy nosných raket

Guidelines:

The objective of this work will be gathering relevant information on recovery technology of launchers and how this technology can be applied for cubesats and the impact this development can have in the space sector.
1) Research and compare various existing launcher recovery technologies.
2) Study compatibility of launcher recovery vehicles with cubesats
3) Develop an example draft for prototype propulsion with recovery system
4) Study the impact and importance of developed concepts in the space sector.

Bibliography / sources:

[1] Patrick Gallais, "Atmospheric Re-Entry Vehicle Mechanics," Springer-Verlag Berlin Heidelberg 2007
[2] Mohammad Sadraey, "Unmanned Aircraft Design: A Review of Fundamentals," in Unmanned Aircraft Design: A Review of Fundamentals, Morgan & Claypool, 2017

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III. Assignment receipt

The student acknowledges that the master's thesis is an individual work. The student must produce his thesis without the assistance of others, with the exception of provided consultations. Within the master's thesis, the author must state the names of consultants and include a list of references.

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Date of assignment receipt	Student's signature

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Declaration

I declare that the presented work was developed independently and that I have listed all sources of information used within it in accordance with the methodical instructions for observing the ethical principles in the preparation of university thesis.

Date and place : Prague, August 14, 2020

Signature:

Mauro Eusebio Rojas Sigala

Acknowledgment

Dedicated to my family and friends who have supported me during this great experience.

Abstract

The space sector has been evolving due to the fast-technological advancements generating a reduction of manufacturing, cost and size in space missions, where highly capable performing small satellites are becoming the standard in this industry. Furthermore, the high launching cost limits the trend of cost reduction for the space missions, since the small satellites are sent as a second payload. An alternative to reduce this limitation is using reusable launchers which are key in the future of space industry, once they are optimized in efficiency and reliability. Therefore, an opportunity of design is presented, since the increase of small satellites missions requires a reduction of the cost in launch services a suitable option for the future market are the reusable launchers. The problematic of using recovery systems and reuse parts of the vehicle is the increase of weight due to the added systems that the vehicle needs to be recovered.

This paper presents different engines and calculate the performance of each engine based on the needs of missions for small satellites. The starting conditions will be that the payload needs to be launched in low circular or elliptical orbits (altitudes of between 300 and 650 km) and the engine has the ability of vertical take-off, vertical landing. The design will also take into account the possibility of reusing parts of the vehicle and the reentry capability. Different combination of engines and fuels are setup in various configurations. For each case the mass analysis will be developed which will allow to calculate the performance for each engine. The important parameters are the number and type of engines, the ratios of the masses, the thrust-to-weight ratio and specific impulse. Once the mass analysis is obtained the following procedure is the selection of the design considering the empty mass. The best combination of characteristics of the engines will be the suitable candidate. Different assistance systems and techniques for the recovery are assessed to obtain a suitable option to improve the efficiency. The expected results are the calculation of the engine performance and how the selected design can be suitable for the space launcher sector for the small satellites. The expected results are a feasible vehicle for small satellites design based in the calculation of the engine parameters together with an efficient launch recovery system. The conclusion is that the space sector can benefit from the design, demonstrating that a launch vehicle with the reusable characteristics can deliver small satellites as a primary payload in a safe, reliable and relative affordable mission.

1. Introduction

The international market for space has advanced and grown over the last decade. From an estimation of $176 billion in 2006, the worldwide area market has improved to an estimate exceeding $345 billion in 2018. One of the modifications in the sector is due to enhancing performance and efficiency by reducing the price, this also includes pushing forward and selling new, privately owned areas organizations and companies stimulating a market where competitors are able to enter the marketplace, this transition generates a change where "price-plus" contracts are less common than "fixed-price" contracts. Cost-plus refers to an association or company that charges all the cost of doing a project and on top of that also includes an extra charge to obtain an income or an additional profit. Cost-plus projects are suitable whilst the extent for an attempt required to complete it is unknown. Compared to a price-plus system in which most of the cost of the bill, to develop such project, is planned and performed based purely in the requirements and no extra profit. This makes the company and its team to prevent possible dangers by developing a proper plan to deliver the product/project at the scheduled time with almost no margin of error. This increment of responsibility to the contractor presents an incentive to estimate the price and overall development, motivating the contractor to fulfill objectives, and permits for truthful and affordable fee negotiation since the very beginning. (NASA, 2019)

Since the space market is changing over the past years, one of the areas of interest is the launcher market. During the last decade there has been a lot of interest on reusability systems launchers (RLVs) used for launching payloads into space. This idea comes to obtain a reduction of the cost by reusing the full vehicle or part of the vehicle, instead of being lost in the way. If achieved properly there will be a drop eventually through multiple uses and missions. (Kelly, 2020). The idea in practice of saving and reusing the elements of the vehicle of a space mission has always being consider since the first research or rocketry and launch vehicle.

The main problem for shared missions is the schedule, delays and the fact that the small payload customer has to wait for the bigger payload customer, usually the main one, to be ready for launch. Of course, if the small payload user plans to use an exclusive launcher system the price is usually out of range of their budget for the mission. (Etherington, s.f.)

2. Literature Review

2.1 Recovery and Reuse Techniques

The main idea for the action of recovering hardware from a space mission includes several actions; atmospheric reentry, deceleration, and landing. The reentry action can be done either by via retro-propulsion, or by taking advantage of the atmosphere to reduce velocity of the vehicle enhancing the aerodynamic drag, retropulsion is also a way of deceleration. The landing can be obtained with the assistance of a type of landing gear or with an assist of third-party systems where the element lands in water or another type of vehicle. This section presents a brief description of the following techniques.

Retro-Propulsion

The Retro-propulsion has gained a lot of popularity in the last years, is also the most straight forward method of all, the concept is using the propulsion system to control the reentry and land. The first successful attempt in history was done by the Apollo program to land man on the moon in 1969.
The vehicle with a retro propulsion system is a combination of multiple systems to obtain this objective. Once the rocket starts its decent the altitude and control system start giving feedback and action to the sensors then the system starts controlling the vehicle accordingly to its environment. Space X has developed one of the most efficient recovery systems ever, and it is mounted in its rocket Falcon 9. As an example, this rocket has a cold thrusters control system to maneuver it at reentry flight, once the descent starts some of its engines are turned on and off accordingly to achieve a safe landing. It also includes legs that deploy from the main frame, developed since 2013 with a project called Grasshopper. (SPACE X, 2020)

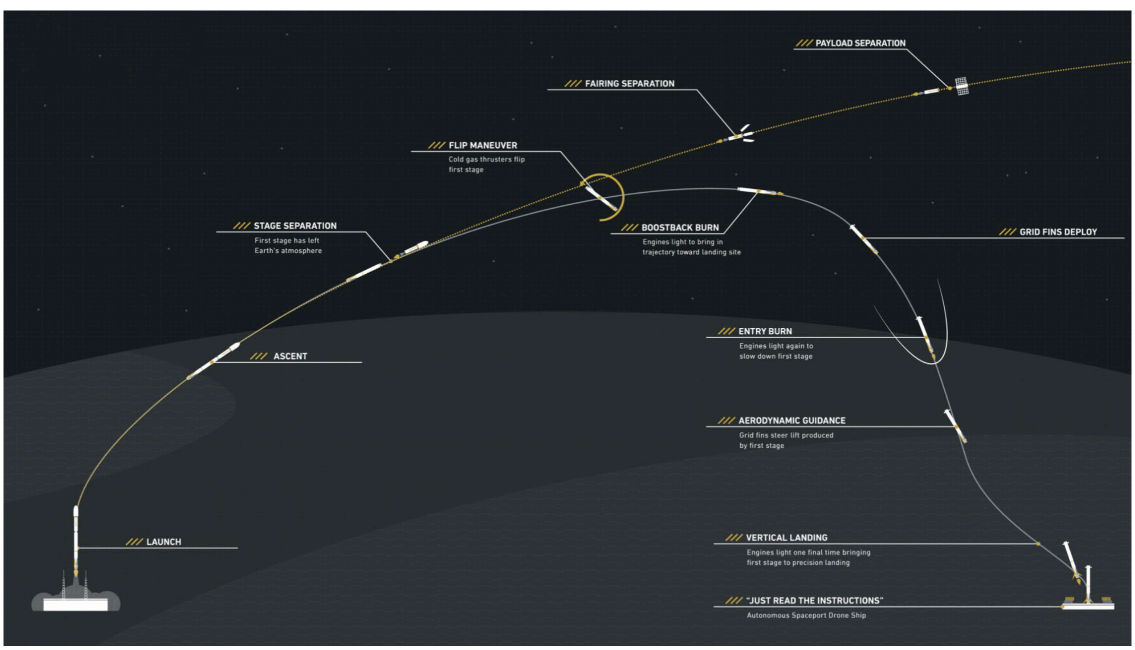


Figure 1. Falcon 9 Sample mission profile and reentry maneuver. (SPACE X, 2020)

Return Flight of Orbital or Suborbital Vehicles

The return flight was first done by the space shuttle orbiter. The man advantage of the return flight compared to the retro propulsion can be consider when the vehicle uses the atmospheric conditions as an advantage to start descent and land, since the design of a vehicle using this technique is more aerodynamic than the shape of a rocket. The return flight orbital vehicle has the capability to return to the exact same position from where it deployed, but it is affected by the weather constrains. Even though this option is flexible it comes with a cost, not only monetary but in terms of the mission. The vehicle has a propulsion system very specific to its kind, the surface has to be aerodynamic and also include a thermal protection. This extra systems and specific requirements are the reason that NASA did not peruse this project due to its complexity and cost.

Recovery of Launch Vehicle Components

To recover a part of a vehicle or the actual vehicle there are external recovery systems such as the Hypersonic inflatable aerodynamic decelerator (HIAD) technology. The general idea is a parachute that withstands extreme conditions. It deploys during the descent for the reentry for a planet or even moons. The HIAD technology is stored in a shroud that has the diameter of the vehicle and it is constraint to aeroshells. The system is light weighted, has an efficient process to be installed in the vehicle, can slow down the reentry object from subsonic and hypersonic speeds. The downfall of this option is the redesign of the vehicle to incorporate this technology also in order to achieve the full recovery it needs the support of different recovery systems depending on the mission requirements, like surface control and some type of propulsion.

Landing Impact Attenuation

The landing impact attenuation system is used in special payloads and as its name says it uses rapidly vented airbags to largely reduce the impact due to acceleration. This system has been largely using in the automotive industry and has been implemented in several aerospace projects for air drop cargo delivery, space capsule landing and even helicopter crash impact attenuation. Once the system starts the compressor releases the gas rapidly onto the system where it inflates the airbag to a very prescribed pressure calculated depending on the requirement of the mission. When the bag impacts it is released through a burst disc venting the gas through a hole and that way reducing the kinetic energy

Mid-Air Recovery

The Mid-Air Recovery technique was developed in the 1969s and used to recover payloads from space for the Corona project. Recent developments in the technology have demonstrated a technique that is reliable end we use to recover up to 10 pounds of payloads. The first stage of the mid-air recovery uses a parachute to reduce the acceleration of the payload. Then a helicopter with an articulated grapple captures the payload by the parachute, through a stable and predicted descent velocity previously calculated. This step transfers the load of the parachute to the helicopter. Finally, the helicopter takes the payload and to a precise location where it can be at land, sea and even a barge or ship for the final recovery. The mid-air recovery is to avoid a landing at sea water and is not subjected to a high impact acceleration compared with other methods. This benefit comes with a cost since it has several uses of different vehicles and logistics. (Ragab, 2015)

2.2 Reusable Launcher Vehicles from Companies

The following section presents a brief overview of the current companies developing the RLV technology.

DLR CALLISTO

CALLISTO is a reusable demonstrator designed to perform a non-vertical flight with large attitude changes during its nominal reference flight. A mission of this rocket contemplates a propelled ascent phase with final conditions similar to those of an operational launcher in tern of flight path angle and dynamic pressure. A large change of attitude followed by a boost to modify the direction of the velocity vector to enable a return towards the launch range. An optional re-entry boost to decrease the re-entry velocity, an aerodynamically controlled and guided re-entry with transition from supersonic to subsonic regime and a landing boost allowing touch down on the targeted landing area with accuracy and a low velocity which can be absorbed by the landing system. The vehicle then reaches a stable position on the ground and can be passivated. (DLR , 2020)

SPACE X

Space X developed the rocket Falcon 9 and Falcon Heavy launch vehicles the company presents this two options for different missions. Space X can deliver payloads to a wide range of inclinations and altitudes, from low Earth orbit to geosynchronous transfer orbit to escape trajectories for interplanetary missions. The ability to reuse the Falcon 9 rocket was one of the milestones in the company. Space X perfected this system by reusing the first stage of the rocket in 2017. Space X has reflowed 35 rockets as per the date this document was written. By reusing the recovered rockets Space X reduces cost and in order to continue doing it is developing procedures to inspect hardware and incorporating new designs that improve the mission from previous flights information.

BLUE ORIGIN

Blue origin developed the rocket New Shepard which is powered by the robust BE-3 engine and is expected to deliver astronauts to space. At full throttle, BE-3PM generates 490 kN (110,000 lbf) of thrust level. Once returning to Earth, it reignites to ninety kN (20,000 lbf) decreasing the velocity to a mild vertical landing on the pad. The main engines, BE-3PM uses high performing LOX and liquid oxidizer for propellant. BE-3PM is intended for operational reusability with bottom maintenance between flights. One of the main objectives are increasing space flights while reducing operative prices. The return system has many key parameters. The main ring and control fins are used during the rocket reenter to the atmosphere, then air flows through the ring at the highest point of the booster, passively moving the center of pressure to descent successfully. The four fins deploy to boost stability and control. While going down additionally the system deploys eight giant drag brakes, reducing vehicles speed. Aft fins help stabilize the vehicle throughout the fall and steer back to the landing descent. Finally, the landing legs are deployed. (BLUE ORIGIN, 2020)

RLV PROTOYPES

Smaller companies are starting to develop some RLV prototypes and here are some of them that present different concepts from the others.

- The company PLD SPACE developed MIURA 1 a rocket with a length of 12.7m and 0.6 m of diameter powered by TEPREL-B engine. The rocket can fly with a payload of as much as 200 kg in a suborbital trajectory. On its first mission, it is expected to deliver 100KG kg of payload to an orbit of 150 km. Additionally, MIURA 1 is ready with a recuperation system that permits PLD Space to recover and reuse the whole vehicle. With this, it'll be the primary recoverable reusable rocket in Europe. The company is also developing the MIURA five (previously referred to as ARION 2) is a 23m length rocket with two stages is planned to deliver 300kg of load in a 500km Heli synchronous orbit. The propellant that the vehicles use is liquid-powered, and the first stage of Miura 5 is designed to reuse the first stage multiple times, once the technology used in MIURA 1is fully tested.

- ROCKET LABS developed the rocket Electron a 17 m length rocket with a 1.2 m diameter. Electron was not developed with a reusable system, but the company is looking for methods to make them reusable. Rocket labs is considering using the mid-air recover technique which was previously explained. The main components considering this technique will be the use of the parachute a helicopter and due to their landing destination and mission requirements a cargo ship will be the final spot where the recovery mission will take place. (Labs, 2019)

- The company PANGEA AEROSPACE is currently designing and testing the MESO rocket a reusable rocket planned for LEO mission for a payload of 150 kg. The proposed fuel is a liquid propellent and will have 2 stage rocket using an aerospike engine. The aerospike engine is still in development and the main difference between a normal bell shape and aerospike nozzles is that the aerospike does not constrain the flow, leaving it free to expand according to the external ambient pressure. In theory , an aerospike is optimally expanded at every altitude which should be an engine with 10-15% more efficient compared with the traditional bell nozzle. The recovery system contains an electric ducted fan this is activated during the descent and will assist to obtain a soft and successful landing, this removes a part of the stress on the main engine and assist in the control of the landing .

2.3 Selection of Reusability Method

As stated in the abstract the main goal to achieve is study the impact of an RLV that can fulfill the requirements for CubeSats and smaller satellites without being a second payload. This RLV should be an affordable option so this missions can be viable in a near future. Furthermore, our main design driver is affordable cost compared with the current market options. Having the objective clear a decision of not further investigate certain options were made. The following objectives were made based on the previous research.

- A suborbital vehicle that returns through a suborbital flight has been previously developed and according to NASA records and reports such as “Reusable Booster System Review and Assessment” and “To reach a high frontier, a history of launch vehicles” this optional has not been successful or has a very high development cost, thus this option is not suitable for the goal of this research.
- In the case of partial recovery, as Space X has done, is a valid option to pursue by scaling down a Falcon 9 to accomplish the objective of a small RLV. This option seems like a feasible option since this company has done multiple return flights with success. This option can be developed furthermore with different number of engines, different types of engines, and fuel.
- Since there are several companies in the market developing small rocket launchers, the option of adapting one of the recovery systems to an existing small rocket launcher is another goal for the research. For this case the project SMALL INNOVATIVE LAUNCHER FOR EUROPE is consider adapting one of these systems, since its mission requirements are very similar to the ones done in this research.
- Once the prototypes are designed a financial comparison with current missions should be done to asses if the designs are suitable for the space sector and an assessment of the mission cost like the one for Rocket labs can be assessed as well.

Proposed Engines

The following engines will be taken into account for the design of the vehicle.

- The Merlin engine is developed by Space X and powers the first stage of Falcon 9. The fuel is liquid oxygen and a rocket-grade kerosene oxidizer. One Merlin engine emits 845 kilonewtons (190,000 pounds) of thrust at Sea level and a specific impulse of 311 sec, while rising to 914 kilonewtons (205,500 pounds) at vacuum with a specific impulse of 345 sec. This engine also has an area ratio of 16, a chamber pressure of 108 bar, and it has the ability to reignite and throttle capability.

- The Rutherford engine used in the electron rocket uses for fuel LOX/ RP-1 same as Merlin engine. The thrust at sea level is 26 kN and nine engines, with a specific impulse of 303 sec, and the thrust at vacuum is 25 kN with a specific impulse of 311 sec. This engine was not design with a reusability system, but the mission requirements are very similar to the one presented in the thesis.

The Launcher Engine-2 and SMILE PROJECT ENGINE were considered for this section but due the lack of flights and early developments the design will be based on the Merlin engine and both Merlin and Rutherford will be used for validation process.

2.4 Propulsion Theory

Tsiolkovsky Equation

$$\Delta V \; = \; Ce \ln\left(\frac{m0}{mf}\right)$$

the "delta-V" or change in velocity, characterizes both the performance of a rocket vehicle and the performance required to fly on some trajectory or complete some maneuver. Ce, the effective exhaust velocity, characterizes the efficiency of the engine. The ratio m0/m1 is the ratio between the mass of the vehicle at the start and end of the burn, the difference being the mass of propellant consumed.

Thrust

$$F_T \; = \dot{m}_p C_E$$

Where $\dot{m}_p$ is the mass flow rate, the higher the effective exhaust velocity, the lower the mass flow rate required to produce a given thrust. The effective exhaust velocity depends primarily on the propellant chemistry, the design of the engine, and the ambient pressure. Rockets perform less well at sea level than in a vacuum, since the ambient pressure limits expansion through the nozzle.

The weight-specific impulse is

$$C_E \; = I_s \; g_0$$

where g0 = 9.80665 m/s2 is the standard gravity. Specific impulse has dimensions of time and is given in seconds. Engine performance is usually quoted in terms of specific impulse. This avoids the trouble of dealing with different unit systems. The rocket equation may be rearranged into the form.

$$\frac{m0}{m1} = \exp\left(\frac{\Delta V}{C_E}\right)$$

which provides the mass ratio needed to achieve some ΔV given some Ce. It is also useful to define a stage propellant mass fraction

$$\zeta = \frac{mP}{mE + mP}$$

where mE is the empty mass of the stage and mP is the mass of propellant it carries.
Including losses, the ΔV required to achieve a low-Earth orbit is on the order of 9200 m/s and this is the bear with no provisions for recovery, performance reserve, etc.

A reusable two-stage vehicle will require additional delta V and hardware (thermal protection, landing gear, etc.) for recovery, and should be capable of carrying at least some payload.

These requirements increase the complexity and required mass ratio/propellant mass fraction further.

Steps for designing a Launch System

The launch system is defined as a vehicle consists in one or more stages assisted with infrastructure in the ground. It carries a payload into a desired orbit protecting it from the environment and the conditions of ascending while increasing its velocity until the end of the trajectory. According to (Wiley J. Larson, 2005) this are the several considerations and steps in the design a of launch vehicles and upper stages.

1. The first step: collect requirements and constraints, which depend on the mission operation concept. Consider the deployment strategy. Considerations
   - Number of spacecraft per launch
   - Spacecraft dry weight
   - Spacecraft dimensions
   - Mission orbit
   - Mission timeline
   - Funding constrains

2. Identify and analyze acceptable configurations for the launch systems , Identify mission requirements Considerations
   - Weight of spacecraft propellant
   - Orbit -insertion stage weight , if required
   - Weight of booster adaptor
   - Performance margin available
   - Reliability
   -
3. Launch system in spacecraft design.
   - Boosted weight capability
   - Performance margin available

- Reliability
- Schedule vs. vehicle availability
- Launch availability

4. Determine spacecraft design envelope and environments dictated by the launch system selected. Include the following considerations and worst-case scenario
    - Fairing size and shape
    - Maximum accelerations
    - Vibrations frequencies and magnitudes
    - Acoustic frequencies and magnitudes
    - Temperature extremes
    - Air cleanliness
    - Orbital insertion accuracy
    - Interfaces to launch site and vehicle

3.0 Methodology

The mission is to design a reusable launcher vehicle that can carry a payload of 500 kg or 150 kg to a LEO orbit and has the ability to return the first stage. One of the key requirements for the design of the vehicle is ΔV required for the mission. The information of the necessary escape velocity to a LEO orbit at 300 km was obatained from previous missions and it is concluded that is approximately $9400\, m/s$. (Wiley J. Larson, 2005).

3.1 First Iteration of Prototype Design

Change of Velocity Calculation

Now taking the design and data from the Falcon 9 a smaller design is going to be developed to fit the requirements for our mission. The following data was taken for the most amount of payload the rocket can carry to a LEO orbit. The first step was obtaining the ΔV for both of the stages. Where the Tsiolkovsky equation, equivalent to the one expressed before, was used.

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln\left(\frac{m_0}{m_f}\right)$$

The following equations were used to calculate the total masses of both stages.

$$mass_{Total} = mass_{Inert\ Structure} + mass_{Propellant} + mass_{payload}$$

Where the total change of velocity is presented in the following equation.

$$\Delta V_{\ Total} = \Delta V_{\ Stage\ 1} + \Delta V_{\ Stage\ 2}$$

Results

Stage 1 Information Falcon 9	**Data**
Inert Structure mass	19,000 Kg
Propellant mass	414,000 Kg
Engine mass (9)	470(1 engine) x 9 = 4230 kg
Specific Impulse	310 s
Payload	22,800 Kg
Change of Velocity Stage 1	4051.7 m/s

Stage 2 Information Falcon 9	**Data**
Inert Structure mass	4,900 Kg

Propellant mass	97,000 Kg
Engine mass	470 kg
Specific Impulse	345 s
Payload	22,800 Kg
Change of Velocity Stage 2	5091.5 m/s

$$\Delta V_{\ Total} = 9260.1\frac{m}{s}$$

The information from previous missions show that the total velocity is 9400 m/s for our mission does the Delta V required for Falcon 9 mission is approximately 9260.1 m/s this means that this specific Delta v is probably for a lower orbit. But this is a good preview of what the impulse in our rocket would need.

Based on this calculations the change of velocity required for stage 2 will be of 5200 m/s and for stage 1 it will be 4200 m/s.

Design of Rocket first iteration

The first step into designing an engine is to establish the requirements which it needs to fulfill, and the constraints imposed. These will define the key parameters needed for the design. The requirements of the system are related to the mission, environment, purpose, reusability or reliability and the constrains to allowable mass or volume.

1. Two stage of an existing multistage vehicle. Continuous firing and vacuum operation.
2. Low cost and high performance (Isp)
3. Payload mass of payloads = 500kg
4. Velocity increase of Delta V = 9,400m/s to LEO
5. Propellant selected is Liquid oxygen-RP1

Propellant

One of the important parameters to consider in the design of a vehicle is the type of oxidizer and fuel that will be used. The Falcon 9 uses a liquid bipropellant of liquid oxygen-RP1. More information can be obtained from the following figure which was developed Based on the performance analysis as a function of mixture ratio. (GEORGE P. SUTTON, 2017)

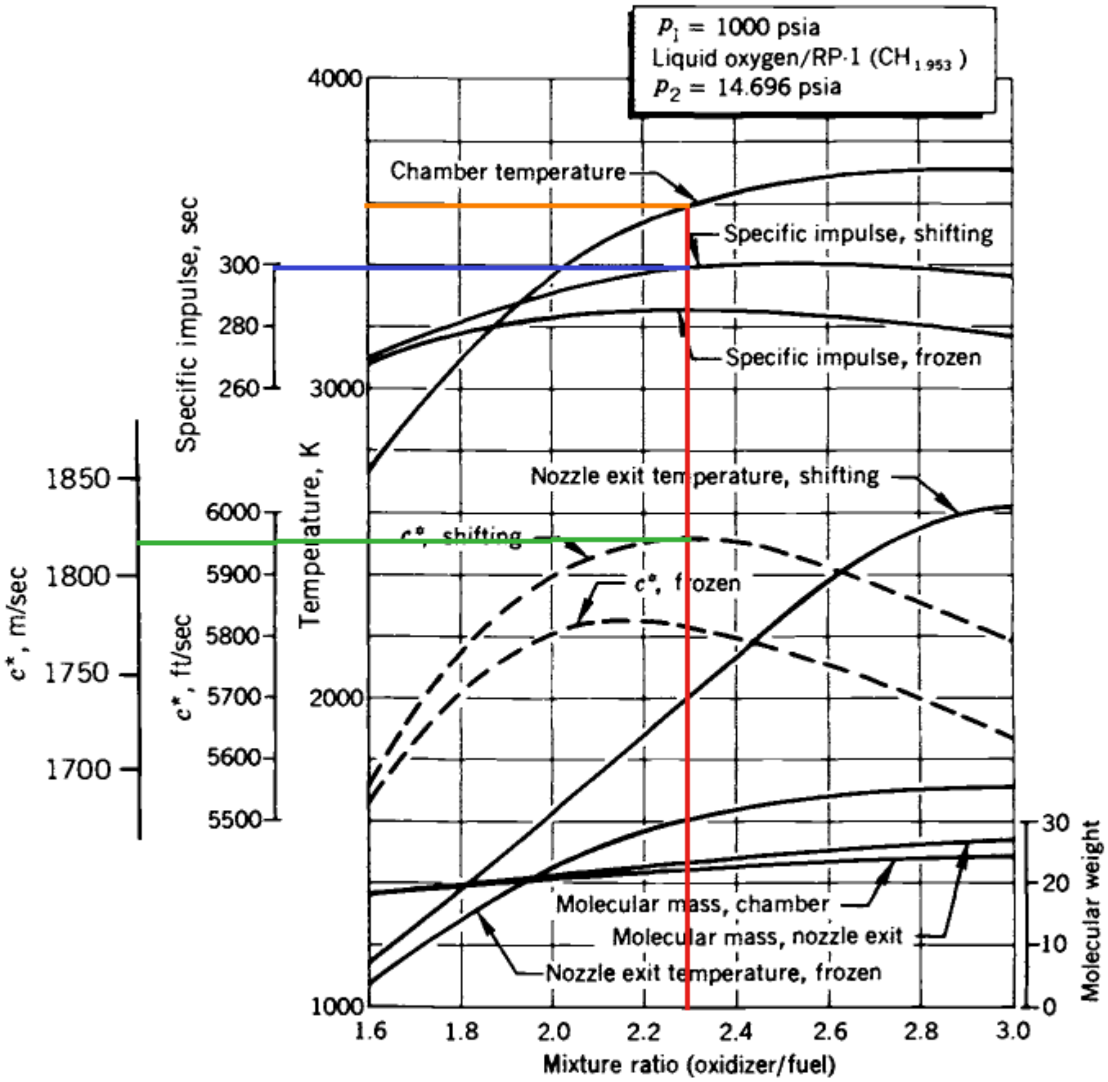


Figure 2. Temperature, Mixture ratio graph for LOX RP-1 (GEORGE P. SUTTON, 2017)

Where the following data is obtained:

- Characteristic velocity, c*= 1820 M/S.
- Ratio of specific heats, k= 1.24.
- The mixture ratio, r= 2.3.

From the decision of the fuel the density of the fuel and density of the oxidizer needs to be obtained as well as the average density. (Wade, Lox/Kerosene, 2019)

The first calculation done in this section will be for the mass propellant based on the Tsiolkovsky equation and designing rules . The following equation explains the mass propellants in terms of the payload mass, since this is a known parameter for our mission. For rocket design usually the inert mass of the rocket is

7% of the propellant mass, in the case of the Falcon 9 stage 2 is approximately 6%, but we will follow the most common designing rule which is 7% for our case. (GEORGE P. SUTTON, 2017) (SPACE X, 2020).

$$\exp\left(\frac{\Delta V}{I_{sp} \cdot g_0}\right) = \frac{m_0}{m_f} = \frac{m_p + 0.07m_p + m_{payload}}{0.07m_p + m_{payload}}$$

$$m_p = \frac{m_{payload}[\exp\left(\frac{\Delta V}{I_{sp} \cdot g_0}\right) - 1]}{1.07 - 0.07\exp\left(\frac{\Delta V}{I_{sp} \cdot g_0}\right)}$$

The initial and final mass are also calculated.

$$m_{structure} = 0.07 * m_p$$

$$m_{total} = m_p + m_{structure} + m_{payload}$$

The following step was calculating the mass flow rate. Where F will be the thrust I_{sp} the specific impulse and g_0 gravity. Where In the equation it can be seen that as the thrust increases also the mass flow rate increases which will mean a bigger thrust chamber.

$$\dot{m} = \frac{F}{c} = \frac{F}{I_{sp} \cdot g_0}$$

Once the total mass flow is calculated the next step will be obtaining the mass flow of the fuel (m_f) and for the oxidizer(m_o).

$$\dot{m_f} = \frac{\dot{m}}{r + 1}$$

$$\dot{m_o} = \frac{\dot{m} \cdot r}{r + 1}$$

The gas generator is considered where it will consume approximately 2% of the total flow.

$$\dot{m_{fgg}} = 0.2 \cdot \dot{m_f}$$

$$\dot{m_{ogg}} = 0.2 \cdot \dot{m_o}$$

Next the thrust chamber flow is solved with the following equations.

$$\dot{m_{ftc}} = \dot{m_f} - \dot{m_{fgg}}$$

$$\dot{m_{otc}} = \dot{m}_o - \dot{m_{ogg}}$$

Duration is calculated with the following equation which is total effective propellant mass divided by mass flow rate of the propellant.

$$t_b = m_p/\dot{m}_p$$

Dimensions

In the following section some calculations are done using data from the fuel liquid oxygen / RP- 1 (Wade, Astronautix, 2019), the data of this fuel and it's oxidizer was obtained as well. The parameters that will be estimated in this section will be done for the dimensions of the thrust chamber. The first parameter that we can obtain will be the area throat A_t. Where C_F is the thrust coefficient and p1 is the pressure at the chamber.

$$A_t = \frac{\mathrm{F}}{C_F \cdot p1}$$

The thrust coefficient can be calculated using characteristic velocity, specific impulse and gravity as follows.

$$C_F = \frac{I_s \cdot g_0}{c^*}$$

The nozzle area ratio is taken from the data of the Merlin engine 1D which is the one Falcon 9 is using, ends from this parameter A2 or also known as exit area can we obtain.

$$e = \frac{A_2}{A_t}$$

Recalling the isentropic relationships and with the data of the area ratio at the throat, the Mach number at the exit can be obtained with the following equation.

$$\frac{A_2}{A_t} = e = \frac{1}{M_2}\left(\frac{2 + (k-1) \cdot M_2^2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

Once the Mach number at the exit is obtained, we can also get the pressure at the exit, p2.

$$p2 = \frac{p1}{\left(1 + \frac{k-1}{2} \cdot M_2^2\right)^{\frac{k}{k-1}}}$$

The next step is calculating the total volume which will be the mass of the propellant divided by the average density of the propellant.

$$V_t = m_{propellant}/\rho_{average}$$

When the total volume is obtained the volume of the tanks for the fuel and for the oxidizer can be obtained using their own densities (Wade, Lox/Kerosene, 2019). The following equations will be calculating these parameters where r represents the mixture ratio.

$$r_v = r \cdot \frac{\rho_f}{\rho_o}$$

$$V_{t,p} = \frac{V_t}{r_v + 1}$$

$$V_{t,o} = \frac{V_t \cdot r_v}{r_v + 1}$$

Then the length of each cylinder tank for the fuel and oxidizer are calculated as shown. A shape of a cylinder is more favorable for the design of the tanks a sphere shape was done before, but it occupies more space. The diameter of the tank was taken from similar rockets with the similar size and mission diameter of the tank is ≈ 1m. (GEORGE P. SUTTON, 2017) (Labs, 2019) (Labs, 2019)

$$L_{t,p} = \frac{V_{t,p}}{\pi\sqrt{D_{tank}}}$$

$$L_{t,o} = \frac{V_{t,o}}{\pi\sqrt{D_{tank}}}$$

The length of the tanks of the fuel and oxidizer need to have an increase of 6% considering fuselage and structure.

$$L'_{t,p} = 1.06 * L_{t,p}$$

$$L'_{t,o} = 1.06 * L_{t,o}$$

The chamber diameter should be about twice the nozzle throat diameter to avoid pressure losses in the combustion chamber.

The nozzle is a bell-shaped with 80% contour of that of the conical one. Assuming a cone half angle of 15° , the nozzle length (L_2) can be estimated according to

$$L_2 = 0.8 \cdot \frac{d_2 - d_t}{2 \cdot tan15º}$$

The following equation is used to determine the chamber volume.

$$V_c = \frac{\dot{m} \cdot t_s \cdot R \cdot T_1}{p1}$$

Where the unknowns are T_1 and t_s. The temperature at the chamber is calculated with the isentropic relationships with the assumption that $v_1 = 0$. The residence or stay time, the average duration of a propellant particle inside the pressure chamber. This parameter is usually obtained after analyzing the flow with CFD software Does the following calculation is beyond this report . The residence time typically varies between 1ms and 40ms, depending on combustion process, chamber size etc. Since the choice of the residence time is difficult to estimate, a value which result in typical volume values for small chambers is chosen ≈ 2ms.

$$T_1 = T_{01} = T_{02} = T_2 \left(1 + \frac{k-1}{2} M_2\right)^2$$

$$T_2 = \frac{k}{R} \left(\frac{p_2 \cdot A_2 \cdot M_2}{\dot{m}}\right)^2$$

$$Vc = \dot{m} \cdot V_{average} \cdot t_s$$

Next the chamber volume is obtained the length of the chamber can be calculated, obtaining the area of the chamber. The one point 1 factor represents a 10% of the chamber volume so that there is enough space for a proper combustion.

$$L_c = 1.1 * \frac{V_c}{A_c}$$

The term L_i represents the length of the injectors and valve system in the calculation of the total length.

The term residence time is difficult to visualize and estimate without the proper tools. Therefore, the customary method of establishing the L_c of the new thrust chamber largely relies on past experience with similar propellants and engine size. Under a given set of operating conditions, the value of the minimum required L_c can only be evaluated by actual firings of experimental thrust chambers.
The throat size of the engine was obtained with a fair degree of confidence. Therefore, historical data is plotted comparing the relation between the chamber length and the throat diameter. plots this relation with an approximating equation obtained from experience. (Braeunig, 2012)

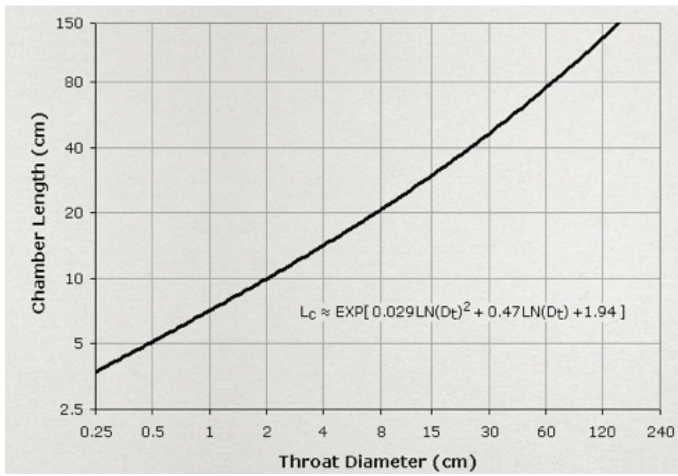


Figure 3 Fit of experimental data of chamber lengths respect to the throat diameter

Results

The results were done using two MATLAB codes developed by the author of the thesis. The codes were STAGE_2_Design.m and STAGE_1_DESIGN.m

From Merlin engine 1D.

Parameter	**Value**
Propellant	Liquid oxygen and (RP-1)
Specific Impulse	345 seconds
Pressure Chamber	10.8 MPa
Nozzle area ratio	16
Trust F	981 kilonewtons
Mixture ratio	2.3
Molecular mass	23 kg/mol
Characteristic velocity c*	1820 m/s
Gas constant R	8314 J/kg-mol-K
Tank diameter	1 m
Length of injector and valves Li	0.30 m
Density fuel	806 kg/m^3

Density oxidizer	1141 kg/m^3
Density average propellant	1014 kg/m^3
Specific heat k	1.23

The following section presents the results of the calculations done based on the propellant selection obtaining propellant mass the mass flow rates and burning time.

Parameter	**Result**
Propellant mass m_p	2579 kg
Burning time t_b	8.89 s
Total mass flow $\dot{m}$	289.97 kg/s
Mass flow Oxidizer in gas generator, $(\dot{m}_o)_{gg}$	4.04 kg/s
Mass flow Fuel in gas generator, $(\dot{m}_f)_{gg}$	1.76kg/S
Mass flow Oxidizer in thrust chamber, $(\dot{m}_o)_{tc}$	198.06 kg /s
Mass flow fuel in thrust chamber, $(\dot{m}_f)_{tc}$	86.11 kg/s

The velocities and the thermodynamics state variables are presented.

Parameter	**Result**
Exit Mach M_2	3.71
Exit pressure p_2	66.54 kPa
Throat pressure p_t	6.03 Mpa
Chamber temperature T_1	3922 K
Throat temperature T_t	3518 K
Exit temperature T_2	1514 K

Finally with the relevant data computed the dimensions of the thrust chamber are presented. With some of the data

Engine part	Subscript	Volume [m_3]	Cross Area [m_2]	Diameter [m]	Length [m]
Throat	*t*	~~N/A~~	0.0489	0.2494	~~N/A~~
Nozzle exit	*2*	~~N/A~~	0.7819	0.9977	1.17
Thrust chamber	*c*	0.0761	0.1955	0.4989	0.428527
Injector and valves	*i*	~~N/A~~	~~N/A~~	~~N/A~~	0.30
Oxidizer tank	*t,o*	1.5748	N/A	1	0.5314
Fuel tank	*t,p*	0.963	N/A	1	0.3270
			Total diameter->	1m	2.704

There are several parameters that may not fit adequately to a real structure. The first one will be the actual diameter selected for the tank which is 1 meter and according to the calculation the not so exit it's

0.9977 which it has not a lot of range for the assembly this may be solved by increasing a little bit the diameter to approximate approximately 1.2 meters. Another Amateur work parameter worth mentioning is that Is that burning time it's only of eight seconds this is mainly because the must flow rate is very high this could be fixed by reducing the thrust of the engine.

This was the first design of the thrust chamber for the second stage of a reusable rocket engine I plan to develop a similar methodology for stage one including the reentry requirement and once the methodology is developed also include a design with the two other selected engines. I would like you to verify this methodology as well as my assumptions for the calculations of the required velocity for the LEO orbit that I got from the first section based on the Falcon 9 numbers.

Chamber Length Second Correction

The following results are a second design correcting the parameters from the first iteration.

- Select L_c from Fit of experimental data of chamber lengths respect to the throat diameter does $L_c = 0.4259\,m$.
- Obtain the volume from L_c
- Reduce thrust 5 times to achieve a larger burning time.

Design stage 2

Trust F	16.2 kilonewtons

Parameter	**Result**
Propellant mass m_p	2579 kg
Burning time t_b	44.48 s
Total mass flow $\dot{m}$	57.99 kg/s
Mass flow Oxidizer in gas generator, $(\dot{m}_o)_{gg}$	0.8084 kg/s
Mass flow Fuel in gas generator, $(\dot{m}_f)_{gg}$	0.3514 kg/s
Mass flow Oxidizer in thrust chamber, $(\dot{m}_o)_{tc}$	39.61 kg /s
Mass flow fuel in thrust chamber, $(\dot{m}_f)_{tc}$	17.22 kg/s

Parameter	**Result**
Exit Mach M_2	3.71
Exit pressure p_2	66.54 kPa
Throat pressure p_t	6.03 Mpa
Chamber temperature T_1	3922 K

Throat temperature T_t	3518 K
Exit temperature T_2	1514 K

Engine part	Subscript	Volume [m3]	Cross Area [m2]	Diameter [m]	Length [m]
Throat	*t*	~~N/A~~	0.0097	0.1115	~~N/A~~
Nozzle exit	*2*	~~N/A~~	0.1563	0.4462	0.4995
Thrust chamber	*c*	0.0166	0.039	0.2231	0.4259
Injector and valves	*i*	~~N/A~~	~~N/A~~	~~N/A~~	0.30
Oxidizer tank	*t,o*	1.5748	N/A	1.2	0.3690
Fuel tank	*t,p*	0.963	N/A	1.2	0.2271
Total diameter and length ->				1.2 m	1.8215 m

Design Stage 1

The following section presents the results for the design of the stage 1 of the rocket. The base data for the design of the following engine were taken from the Merlin 1D engine at sea level. Since the engine was developed at sea level the following thrust coefficient formula was used, and the following results were obtained.

Parameter	**Value**
Propellant	Liquid oxygen and (RP-1)
Specific Impulse	282 seconds
Pressure Chamber	10.8 MPa
Nozzle area ratio	16
Trust F	654 kilonewtons
Mixture ratio	2.3
Molecular mass	23 kg/mol
Characteristic velocity c*	1820 m/s
Gas constant R	8314 J/kg-mol-K
Tank diameter	1.2 m
Length of injector and valves Li	0.60 m
Density fuel	806 kg/m^3
Density oxidizer	1141 kg/m^3
Density average propellant	1014 kg/m^3
Specific heat k	1.24
Mass stage 2	3260 kg

The following section presents the results of the calculations done based on the propellant selection obtaining propellant mass the mass flow rates and burning time.

Parameter	**Result**
Propellant mass m_p	16486.71 kg
Burning time t_b	69.71 s
Total mass flow $\dot{m}$	236.5 kg/s
Mass flow Oxidizer in gas generator, $(\dot{m}_o)_{gg}$	3.29 kg/s
Mass flow Fuel in gas generator, $(\dot{m}_f)_{gg}$	1.43kg/S
Mass flow Oxidizer in thrust chamber, $(\dot{m}_o)_{tc}$	161.53kg /s
Mass flow fuel in thrust chamber, $(\dot{m}_f)_{tc}$	70.23 kg/s

The velocities and the thermodynamics state variables are presented.

Parameter	**Result**
Exit Mach M_2	3.71
Exit pressure p_2	64.49 kPa
Throat pressure p_t	6.01 Mpa
Chamber temperature T_1	3478.67K
Throat temperature T_t	3105.95K
Exit temperature T_2	1291.15K

Finally with the relevant data computed the dimensions of the thrust chamber are presented. With some of the data

Engine part	**Subscript**	**Volume [m^3]**	**Cross Area [m^2]**	**Diameter [m]**	**Length [m]**
Throat	*t*	~~N/A~~	0.037423	0.218286	~~N/A~~
Nozzle exit	*2*	~~N/A~~	0.598773	0.873144	0.977586
Thrust chamber	*c*	0.063754	0.149693	0.436572	0.425900
Injector and valves	*i*	~~N/A~~	~~N/A~~	~~N/A~~	0.30
Oxidizer tank	*t,o*	10.064477	N/A	1.2	2.3582
Fuel tank	*t,p*	6.194610	N/A	1.2	1.4515
			Total diameter and lenght->	1.2 m	5.8131

After reviewing the information, receiving assessment from tutelage and comparing the results with real data of current rockets the conclusions were that this results are not valid for the conditions thus another method of mass estimation was used and will be explained in the following sections.

3.2 Reusability Systems Concepts, and Parametrization for Prototype

The next section explains reuse for the stage 1 of the rocket. The reusability systems include a cold gas and nitrogen thrusters for attitude control, four hypersonic grid fins used for three-axis control during the atmospheric re-entry flight where there is no propulsion coming from the engines, and four deployable light weighted landing legs.

The return of the first stage uses the cold gas thrusters for control in the yaw, pitch and roll movements, it starts with a maneuver where the second stage engine plume exits, before reorienting to a first engine position that is maintained beyond the point of apogee. Then a pneumatic system using high pressure helium starts the deployment of the light weighted landing legs for a safe landing in a drone ship controlled through GPS.

Returning to the dense sections of the atmosphere, the first stage of Falcon 9, completes its supersonic retro propulsion burn with 3 engines. Where this are turned up for 20 seconds as high as an altitude of seventy kilometers. The total re-entry burn combined with the drag force in the atmosphere slows down the vehicle from 1,300 m/s to approximately 250 m/s. The following data will be used to estimate the amount of fuel the prototype rocket needs to have.

Heading towards its landing point, the first stage starts the center engine to begin slowing down. The landing speed of only about two m/s and make the final maneuvers and range adjustments to land on the platform. Ten seconds after the landing burn, all four landing legs unfold to set the stage in its final landing configuration in the drone ship just seconds later.

Nitrogen Cold Gas Attitude Control System

The cold gas thruster system is consisting in a pressurized tank with gas, where a valve deploys the gas through a nozzle generating a force of impulse. Since the type of gas in this system is not heated an average specific impulse of the system is 68 seconds.

The cold gas thrusters are reliable, and its simple applications are employed mainly for control and have the following specifications:

- Maintain an orbit through altitude control with the thrust
- The system contains a high-pressured tank, where the pressure is regulated and fed to the nozzle usually controlled by the on-board computer of the vehicle realizing the nitrogen.
- Usually the configurations are two thrusters and one latch valve that is used in case of leakage.
- There are several types of propellant such as helium, argon, or gaseous nitrogen.
- The systems are very reliable despite the low cost to develop.
- The system is relatively safe and does not contaminate a sensitive vehicle.
- Some Materials for cold gas tanks are Aluminum, steel, or plastic.

Some disadvantages are:

- The mass fraction is usually low (0.02 to 0.19)

- Thus he specific impulse and the velocity increments are very small.
- Due to the tank configuration they may take a large volume of space depending on the aircraft and mission. (SUTTON)

Deployable legs

The Falcon 9 rocket has a system of deployable legs for landing safely from reentry. Each of these legs have a telescoping piston connected with two smaller rods in an A-frame shape. It is the materials are carbon fiber and aluminum and the total length of the extensible landing legs is meters, and the weight is less than 2,100 kilograms. For the action of deploying before touchdown, a system of high-pressure Helium is used. It contains a "crush core", for shock absorption where the impact is severely nullified specially in hard landings and to retrieve the vehicle in optimal conditions. The whole system conformed by four legs have a honeycomb shape and are mounted symmetrically at the bottom of the rocket and around its base. The A-frame shape legs are tucked in alongside of the vehicle and they have almost no aerodynamic effect during the trip.

A method of scaling the deployable legs is by obtaining a scaled down factor, F , from the original Falcon 9 and use the prototype mass to obtain this scale factor for the measurements of the legs

$$F = \frac{Falcon\ 9\ Stage\ 1\ mass}{Prototype\ rocket\ stage\ 1\ mass} = \frac{19{,}000\ kg}{1154\ kg} = 16.46$$

$$Mass\ of\ legs\ for\ prototype = \frac{mass\ of\ legs\ for\ Falcon\ 9}{F}$$

$$Mass\ of\ legs\ for\ prototype = \frac{2100\ \text{kg}}{16.46} = 127.58\ Kg$$

$$Mass\ legs\ span\ for\ prototype = \frac{18m}{16.46} = 1.09\ m$$

Approximation of Cost Deployable Landing Legs

For the assessment of the cost deployable landing legs the research done for the following section started with the development of the visible technologist for the first stage booster launch vehicle designs that SpaceX developed uh this three vehicles were the grasshopper , F9R DE V1 and F9RD E V2 . The program that was developed for reusable technologies included several sections. The first started with a low altitude at low velocity testing one single engine this was for the grasshopper technology demonstrator. The second low altitude section Which were which had a higher altitude of three kilometers it was also at low velocity testing but had a larger vehicle to test which included three engine tested for the F9RD E V1 in these steps the landing legs will Lex where already included . The third section was a high altitude at mid velocity testing, but it had to be postponed due to the development of the rocket F9R DEV 2. The final program was a high-altitude test for 91 kilometers which had a high velocity starting from Mach 6 and decreasing all the way to 2 kilometers over seconds this was classified as a ballistic reentry , it had control deceleration and decent tests. The system that was tested also worked in the following stages of the Falcon 9 to test its ability and landing system.

To assess the cost of the landing legs the following actions need to be consider; development and manufacturing, materials and manpower is not an easy task to develop or to obtain which will be consider a high cost to obtain. But the most important part for developing the system is the fact that it has to be done in the similar way of the tests presented. In the case of the grasshopper, eight low altitude flights were made between 2012 and 2013 does the price for developing landing legs will be the development of these tests throughout a year. The first booster return controlled-descent test from high-altitude was made in September 2013, with a second test in April, a third test flight in July and a fourth test in September 2014. All four test flights to date were intended to be over-water, simulated landings. Five low-altitude booster flight tests of F9R Dev1 were flown during April–August 2014, before the vehicle self-destructed for safety reasons on the fifth flight. Finally for a proper economical assessment of the landing legs a similar project will have to be assessed. Where the main components are the structure, engines for a stage one , the fuel for several tests(between 6 to 8), design and development of the landing legs including the material, as well as the team to develop the project and testing. (SPACE LAUNCH REPORT, 2017)

Hypersonic Grid Fins

When the rocket is returning at approximately seventy kilometers of altitude, the thrusters would start three engines to decelerate the rocket, and as the thrusters began to descend, they would use four grid fins to steer and stabilize the thruster for the rest of the descent. The grid fins, which are used at both supersonic and subsonic speeds, can rotate and tilt simultaneously, allowing very precise orientation and precision control during the flight. The fin movements can be performed in a pitch, yaw and roll motion. The grid fins were designed in an "X" configuration, the hypersonic grid fins have the function of controlling the descending rocket's lift vector during the vehicle return to the atmosphere enabling a much more exact landing location. The grid fins are made from forged titanium.

To design a set of fins for the prototype presented in this document, the measurements from the original fins of the Falcon 9 were taken from pictures, the results are presented in the following table, where there are rough approximations.

Falcon 9 Fin	**Size**
Height	1.35 m
Width	1.01m
Depth	0.22 m
Square	0.11m
Width squares (mesh)	0.05m
Total mass (4 fins)	3369.87 Kg

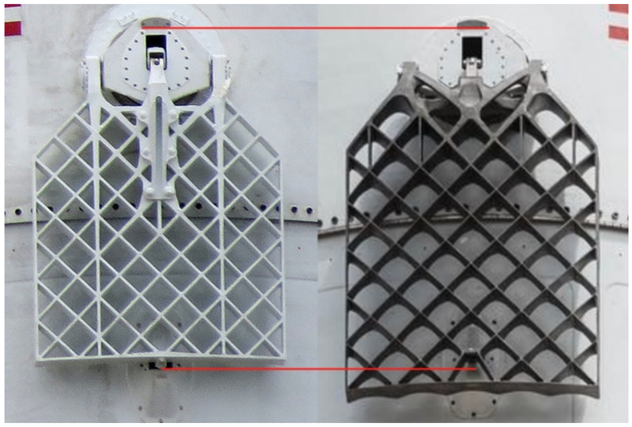


Figure 4. Grid fins Stack exchange, s.f.)

The hypersonic grid fins are usually developed by numerical modeling of the flow field in the specific interface zone of the free stream of a vehicle, where CFD modeling is also used. This process also takes into account using the high-resolution solver for the Navier-Stokes equations and are compared with experimental data of the surface streamlines and heat flux distributions obtained under similar conditions. The design of the grid fins for the prototype is beyond the scope of this study but the weight has to be calculated. To obtain the dimensions of the fin, the materials and the measurements divided by two from the original Falcon 9, will be taken into account to calculate the weight of the hypersonic fins. (Tutty, 1995)

Prototype	**Size**
Height	0.675 m
Width	0.505m
Depth	0.11 m
Square	0.055m
Width (mesh)	0.025m

To calculate the mass of the grid fin, the measurements of only one rod are obtained, where the volume can be obtained and with the density of the titanium, 4510 kg/m^3, the mass of the rod is calculated. Taking into account the dimensions of the fin with the help of the photographic approximation tool, a total of 6.3 rods are calculated to be fitted in one of them with a mass of 16.74 kilograms, and a second set of rods to generate the grid of 8.4 rods with a mass of 12.52 kilograms. Does the total mass of the grid fin is approximately 210.64 kilograms and the total amount of mass taking into account four grids will be 842.56 kg.

A second method of designing the grid fins is by obtaining z a scaled down factor, F , from the original Falcon 9 and the prototype then use this scale factor for the measurements of the grid fins and the weight.

$$F = \frac{Falcon\ 9\ Stage\ 1\ mass}{Prototype\ rocket\ stage\ 1\ mass} = \frac{19{,}000\ kg}{1154\ kg} = 16.46$$

$$Mass\ grid\ fins\ for\ prototype = \frac{mass\ of\ grid\ fins\ Falcon\ 9}{F}$$

$$Mass\ grid\ fins\ for\ prototype = \frac{3369.87}{16.46} = 204.7\ Kg$$

Approximation of cost for the Hypersonic Grid Fins

Production prices for structures of titanium is pricey to refine, process, and fabricate. As shown in figure three, titanium, is far dearer than metal and steel, all told stages of production, together with metal processing, metal bar forming, and sheet forming. In terms of the process value of materials per capacity unit, titanium is 5 times more expensive to refine than metal, and around 10 times dearer than metal to make ingots and fabricate finished product. Of all the stages of production, fabrication is that the one with the highest cost, followed by extracting sponge from ore. (Seong, 2009)

Production Stage	Steel	Aluminum	Titanium
Metal refining	0.4	1.0	5.0
Ingot forming	0.6	1.0	10.7
Sheet forming	0.4	1.0	18.0

Figure 5. Prices of titanium compared with prices of aluminum (Seong, 2009)

Following the image on the production and prices for the titanium, the cost of the grid fins are calculated based on this data. According to the website Indexmundi (Indexmundi, 2020) which takes statistics from the World Bank the aluminum has a price as of May 2020 of $1466 per metric ton Meaning 1 kilogram of aluminum would cost around 1.5 US dollars. Thus a sheet forming of titanium would cost approximately $27 US dollars but depending on the quality of the titanium it could cost up to $60.00 USD per kg. Recalling the mass of the fourth fence which would be 204.7 kg the material for the four things would be in the realm of $12,282 US dollars approximately. But this is a high-grade material, cast titanium, which is a very difficult process to develop, and the waste of material during the manufacturing of the grid fins are not taken into account thus one grid fin likely costs in the realm of $50,000 USD. This number is a rough estimate of the amount of money that the material of the grid fin would cost. Of course in the development of this parts, this price is only on the raw materials. There also need to take into account the engineers the licensing and special equipment and at the same time they'll the labor cost would be added in the manufacture of the actual rocket since the development of other pieces and the control system has to be included in this process.

Reentry Burn

The following section explains the most important data of the Falcon 9 re-entry stage. SpaceX introduced elements to its Falcon 9 to enable the recovery and reuse of the first stage. According to an interview done to the president of Space X, Gwynne Shotwell, the additional weight and power needed to control the first stage's re-entry for recovery and reuse are already embedded in the Falcon 9 design. "There are no major new hardware or cost items needed to accomplish recovery and reuse, and no reason for Falcon 9 performance to degrade, or prices to increase, to accommodate the recovery mission", she said.

According to the article (Selding, 2014) , the Falcon 9 v1.1 vehicle has about 30 percent more performance capability than what is advertised on SpaceX's price and capacity specifications. The extra 30 percent margin is retained by SpaceX, which will use it to accommodate the additional fuel and hardware needed for first-stage recovery operations. This margin was one of the reasons that SpaceX was able to sign its two contracts, with SES of Luxembourg, for the launch of two satellites, each weighing around 5,330 kilograms which is or 10 % more than the vehicle's advertised payload limit. "The vehicle we plan on recovering is the one we are currently using," Shotwell said. "It is already sized. Once we get to where we are recovering and reusing components, the price will come down from right now. It certainly won't go up." (Selding, 2014)

Using the following information is safe to assume that 30% percent of the mass of the rocket is used in the recovery systems.

Re-entry Burn Time of First Landing

This section presents an approximation for the amount of propellant mass used in the Orbcomm mission done by Space X rocket Falcon 9 since it was the first boost back flight with successful landing.

The stage 1 start the engines for 2 minutes 20 seconds and then separation started. The stage 2 started the engine at 2 minutes 35 seconds for a total of 8 minutes burn to achieve a 620 x 660 km x 47 deg orbit.

In parallel the first stage started the descent and while returning the first ignition was for 30 seconds a boost back burn starting at 3 minutes and 50 seconds after launch, then a 20 second reentry burn after 8 minutes of launch in both maneuvers 3 engines were used . Finally a 32 second ignition with only one engine which was the center engine started to safely achieve the soft landing after 10 minutes of liftoff. (SpaceX, 2015)

To verify the reentry mass that the Falcon 9 used, the burning time is used, and the results are presented. Using the following equation.

$$t_b = \frac{m_{propellant}}{\dot{m}}$$

Where,

The total mass propellant of the stage 1 is, $m_{propellant}$ $=414{,}000\ \text{kg}$, and the mass flow rate is $\dot{m} = 298.73\ kg/s$

Log of reentry stage 1		
Time	Action	Boosters
30 s	Boost back	3
20 s	Reentry Burn	3
32 s	Landing Burn	1

Total time = 152 s

$$m_{propellant} = 54{,}368 \ kg$$

Which is approximately 13% of the mass propellant that is used in the Falcon 9 .

3.3 STAGE MASS OPTMIZATION

The following section presents the calculation for the stage masses of the rocket using an optimization method. The calculations were done using the MATLAB code solve5D and Opt_stg written by (Jeronimo, 2020). The optimization of the rocket supports the task of its design, since the main goal of a rocket is to have the most amount of payload mass for the least amount of propellant mass. The code solve_ 5D and Opt_stg, based on the research of (Anand Kumar, 2020), solve the reduction of propellant and increment of payload problem by optimizing the rocket equation and staging concepts. (Anand Kumar, 2020)

The inputs for the code are:
Payload mass
Specific impulse in stage 1
Specific impulse in stage 2
Change of velocity to achieve the orbit
Sigma; factor ratio for structure mass and propellant mass

Sigma comes from equation

$$\frac{m_f}{m_o} = \frac{m_s}{m_s + m_p} + \frac{m_p m_{pl}}{m_0\left(m_s + m_p\right)}$$

After some mathematical manipulation we have sigma as

$$\sigma = \frac{m_s}{m_s + m_p}$$

The mathematical approach of this method starts by using the derivation and mathematical manipulation of the stage mass and rocket equations to maximize the payload mass ratio using a specific combination of propulsion systems to achieve a change in velocity dependent of the orbit. The optimal solution yields into solving Lagrangian function with the Newton-Raphson method. The solutions obtained by this method is the optimal mass distribution for the rocket stages, where in this specific case the following equations are the system evaluated in the code. (Anand Kumar, 2020)

$$\frac{1}{\lambda_1} + \frac{p\beta(1-\sigma_1)}{[\sigma_1 + (1-\sigma_1)\lambda_1]} - \frac{r}{\lambda_1^2} = 0$$

$$\frac{1}{\lambda_2} + \frac{p\beta(1-\sigma_2)}{[\sigma_2 + (1-\sigma_2)\lambda_2]} - \frac{q}{\lambda_2^2} = 0$$

$$R_v + \sum_{k=1}^{2} \beta_k \ln[\sigma_k + (1-\sigma_k)\lambda_k\ ] = 0$$

To validate the mass optimization method the information of the Electron rocket from the company rocket Labs was used. The first step was to obtain the sigma value for the code. The following table presents the values of sigma for such exercise.

	Real Value Electron	Input for MATLAB Estimated
Sigma stage 1	0.093	0.12
Sigma stage 2	0.108	0.12

The sigma value used in the input for MATLAB is a bit higher since the website of rocket labs presents a total payload of 13,000 kg but with no stage masses defined. In space launch report (Rocket Labs, 2019)the stage masses are presented but the total is 12,650 kg probably omitting the structure mass where the payload is stored. For this reason the sigma value for the calculated results are increased.

The results from the optimization code are:

- Total mass
- Stage 2 and payload mass
- Stage 1 mass
- Stage 2 mass

The structure mass is obtained with the following equation

$$m_s = \sigma * (m_s + m_p) = \sigma(stage\ mass)$$

And the propellant mass is obtained with:

$$m_p = m_s - stage\ mass$$

Validation of Electron Real Data and Calculated with Optimization Code

The following table presents the results of the simulation for validating the optimization method using the values of the Electron rocket.

Parameter	**Real Value**	**Calculated Value**
Mass Payload	150 Kg	150 Kg

Isp stage 1	303 sec	303 SEC
Isp Stage 2	330 sec	330 sec
Dv (Change of velocity to achieve 500 Km SUN-SYNC. orbit)	9455 m/s	9455 m/s
TOTAL MASS	12650 kg	12048.03 kg
STAGE MASS 1	10200	10595.22 kg
PROPELLANT MASS STAGE 1	9250 kg	9323.79kg
STRUCTURE MASS STAGE 1	950 kg	1271.43 kg
STAGE MASS 2	2300	1302.8 kg
PROPELLANT MAS STAGE 2	2050 kg	1146.46 kg
STRUCTURE MASS STAGE 2	250 kg	156.33kg

The results obtained from the calculation using the code are similar to the real values of the Electron rocket. This means it is a good approximation for calculation of mass stages for the rocket prototype. The stage 1 date has an error of approximately 8% where the second stage has an error of 42% in comparison with the real values.

Prototype Data Calculated with 500 kg and 150 kg Payload

The following table presents the results of the optimization method using the values of the Prototype rocket for two payloads with 500 kg and 150 kg.

Parameter	Prototype 500kg	Prototype 150kg
Mass Payload	500Kg	150 Kg
Isp	311 sec	311 sec
Isp Stage 2	345 sec	345 sec
Dv (Change of velocity to achieve 300 Km to Orbit)	9400 m/s	9400 m/s
TOTAL MASS	30727.56 kg	9218.26 kg
STAGE MASS 1	26393.9 kg	7918.17 kg
PROPELLANT MASS STAGE 1	23226.6 kg	6967.99 kg
STRUCTURE MASS STAGE 1	3167.27 kg	950.18 kg
STAGE MASS 2	3833.65 kg	1300.09 kg
PROPELLANT MAS STAGE 2	3373.61 kg	1012.08 kg
STRUCTURE MASS STAGE 2	460.04 kg	138.01 kg

The results show a drastic reduction of the mass stages by only reducing the payload mass. By reducing the payload mass the smaller rocket of 150 kg has 70% less mass than the one that has a payload of 500 kg.

Validation of Electron Dimensions

The following table presents the dimensions using the OptiProtoStage1Tanks.m and OptiProtoStage2Tanks.m corrected from the first iteration of design in the methodology section. Where the inputs changed to only adding the mass obtained by the optimization and not the calculated one. This section validates this method by estimating the length of the real Electron rocket.

STAGE 1	**STAGE 2**
Throat Area=0.001087	Throat Area=0.001135
Throat diameter =0.037206	Throat diameter =0.038021
Nozzle exit area =0.017396	Nozzle exit area =0.018166
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.152084
Nozzle exit length=0.166626	Nozzle exit length=0.170275
Thrust chamber volume=0.001852	Thrust chamber volume=0.001934
Thrust chamber area=0.004349	Thrust chamber area=0.004541
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.076042
Thrust chamber length= 0.468490	Thrust chamber length=0.468490
Oxidizer tank volume =5.646754	Oxidizer tank volume =1.251443
Oxidizer tank diameter=1.100000	Oxidizer tank diameter=1.100000
Fuel tank volume =3.475534	Fuel tank volume =0.770254
Fuel tank diameter =1.100000	Fuel tank diameter =1.100000
Oxidizer tank length =3.876611	Oxidizer tank length =0.859141
Fuel tank length =6.298388	Fuel tank length =1.395859
TOTAL LENGHT=11.467525	TOTAL LENGHT=3.151175m
REAL LENGHT = 12.1m	REAL LENGHT= 2.4 m

The method of dimensioning the rocket presents a very solid approximation. The calculated stage 1 dimension shows that it is 5.3 % smaller than the real one. And for the second stage the calculated value is 13.1% bigger.

PROTOTYPE DIMENSIONS WITH 500 KG

The following table presents the dimensions using the OptiProtoStage1Tanks.m and OptiProtoStage2Tanks.m to obtain the dimensions of the Prototype rocket with a payload of 500 kg.

STAGE 1	**STAGE 2**
THRUST= 19KN	**THRUST 22KN**
Throat Area=0.001087	Throat Area=0.001096
Throat diameter =0.037206	Throat diameter =0.037354
Nozzle exit area =0.017396	Nozzle exit area =0.017534
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.149415
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.167288
Thrust chamber volume=0.001852	Thrust chamber volume=0.001867
Thrust chamber area=0.004349	Thrust chamber area=0.004383
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.074708
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =14.178907	Oxidizer tank volume =2.059453

Oxidizer tank diameter=1.100000	Oxidizer tank diameter=1.100000
Fuel tank volume =8.727011	Fuel tank volume =1.267578
Fuel tank diameter =1.100000	Fuel tank diameter =1.100000
Oxidizer tank lenght =9.734107	Oxidizer tank lenght =1.413857
Fuel tank lenght =15.815151	Fuel tank lenght =2.297114
TOTAL LENGHT=26.784374	TOTAL LENGHT=4.646748

PROTOTYPE DIMENSIONS WITH 150 KG

The following table presents the dimensions using the OptiProtoStage1Tanks.m and OptiProtoStage2Tanks.m to obtain the dimensions of the Prototype rocket with a payload of 150 kg.

STAGE 1	**STAGE 2**
THRUST= 19KN	**THRUST 22KN**
Throat Area=0.001087	Throat Area=0.001135
Throat diameter =0.037206	Throat diameter =0.038021
Nozzle exit area =0.017396	Nozzle exit area =0.018166
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.152084
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.170275
Thrust chamber volume=0.001852	Thrust chamber volume=0.001934
Thrust chamber area=0.004349	Thrust chamber area=0.004541
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.076042
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =5.627378	Oxidizer tank volume =0.617834
Oxidizer tank diameter=1.100000	Oxidizer tank diameter=1.100000
Fuel tank volume =3.463609	Fuel tank volume =0.380272
Fuel tank diameter =1.100000	Fuel tank diameter =1.100000
Oxidizer tank lenght =3.863309	Oxidizer tank lenght =0.424156
Fuel tank lenght =6.276777	Fuel tank lenght =0.689132
TOTAL LENGHT=11.375201	TOTAL LENGHT=2.052053

Reusability Version of Prototype Data for 500 Kg and 150 Kg

The following table presents the results of the optimization method including 20% of the propellant and structure for the reusability system, this percentage was an approximation based on the data collected and the results obtained while trying to calculate the mass propellant through the burning time. This increment was included in the sigma calculation and then the optimization codes were used again to obtain the staging of the prototype rocket. OptiProtoStage1Tanks.m and OptiProtoStage2Tanks.m were used.

Parameter	**Prototype 500kg**	**Prototype 150kg**
SIGMA STAGE 1	**0.2667**	**0.2667**

Mass Payload	500Kg	150 Kg
Isp	311 sec	311 SEC
Isp Stage 2	345 sec	345 sec
Dv (Change of velocity to achieve 500 Km SUN-SYNC. orbit)	9400 m/s	9400 m/s
TOTAL MASS	95846.72	28754.016
STAGE MASS 1	83737.0012	25121.1
PROPELLANT MASS STAGE 1	61404.3	18421.20
STRUCTURE MASS STAGE 1	22332.65	6699.77
STAGE MASS 2	11609.71	3482.91
PROPELLANT MAS STAGE 2	10216.32	3064.96
STRUCTURE MASS STAGE 2	1393.12	417.94

As we can see the increment of the reusable system is quite high in comparison with the normal rocket in both payload scenarios. In both cases the rockets increase 3 times the mass just to include the reusability system.

Dimensions Reusable Prototype 500 Kg

The table presents the results for the dimensions of the prototype with 500 Kg.

STAGE 1	**STAGE 2**
Throat Area=0.001087	Throat Area=0.001096
Throat diameter =0.037206	Throat diameter =0.037354
Nozzle exit area =0.017396	Nozzle exit area =0.017534
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.149415
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.167288
Thrust chamber volume=0.001852	Thrust chamber volume=0.001867
Thrust chamber area=0.004349	Thrust chamber area=0.004383
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.074708
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =37.484859	Oxidizer tank volume =6.236653
Oxidizer tank diameter=2.000000	Oxidizer tank diameter=2.000000
Fuel tank volume =23.071650	Fuel tank volume =3.838613
Fuel tank diameter =2.000000	Fuel tank diameter =2.000000
Oxidizer tank lenght =7.784570	Oxidizer tank lenght =1.295181
Fuel tank lenght =12.647709	Fuel tank lenght =2.104300
TOTAL LENGHT=21.667396	TOTAL LENGHT=4.335258

The size of the diameter was selected to be 2 meters since Electron has a diameter of 1.2m and Falcon 9 is 3.6 m, 2m is between this two diameters and so is the mass of this rocket.

Dimensions Reusable Prototype 150 Kg

The table presents the results for the dimensions of the prototype with 150 Kg.

STAGE 1	STAGE 2
Throat Area=0.001087	Throat Area=0.001096
Throat diameter =0.037206	Throat diameter =0.037354
Nozzle exit area =0.017396	Nozzle exit area =0.017534
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.149415
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.167288
Thrust chamber volume=0.001852	Thrust chamber volume=0.001867
Thrust chamber area=0.004349	Thrust chamber area=0.004383
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.074708
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =11.245403	Oxidizer tank volume =1.871035
Oxidizer tank diameter=1.100000	Oxidizer tank diameter=1.100000
Fuel tank volume =6.921461	Fuel tank volume =1.151608
Fuel tank diameter =1.100000	Fuel tank diameter =1.100000
Oxidizer tank lenght =7.720197	Oxidizer tank lenght =1.284503
Fuel tank lenght =12.543122	Fuel tank lenght =2.086952
TOTAL LENGHT=21.498435	TOTAL LENGHT=4.307233

For this configuration the diameter was selected to be 1.1 m as for the ELECTRON since the mass is still close to the ELECTRON. This last option seems to be the best candidate for including the reusability systems in its configuration and is not a huge rocket to develop in comparison with the Falcon 9, this prototype is half its size and it is intended to low cost missions and mainly used for small satellites.

3.4 Optimization of the reusable mass propellent for reentry

The approach selected to optimize the amount of propellant consisted in obtaining first the maximum altitude where the first stage detached from the second stage. Once this altitude is obtained, the remaining fuel on stage 1 can be obtained and consider the one use for reentry. For the validation case a real mission data for Falcon 9 will be used

Calculation of altitude for the first stage

For the following calculation for the altitude was done using the code Example_11_03.m from the book (Curtis, 2010) where it calculates the speed, flight path angle, the downrange distance and the altitude which is the value of most interest. The following equations are solved by Runge-Kutta-Fehlberg method with the code rk45.m.

$$\frac{dv}{dt} = \frac{T}{m} - \frac{D}{m} - g\sin\gamma$$

$$\frac{d\gamma}{dt} = -\frac{1}{v}\left(g - \frac{v^2}{R_E} + h\right)\cos\gamma$$

$$\frac{dh}{dt} = v \sin \gamma$$

$$\frac{dx}{dt} = \frac{R_E}{R_E + h} v \cos \gamma$$

Including a drag force by the following equation with an assumption of a drag coefficient of 0.5

$$D = \frac{1}{2} \rho v^2 A C_D$$

The atmospheric density changes with altitude thus is based using the following equation

$$\rho = \rho_o e^{-\frac{h}{h_o}}$$

Validation of Altitude Calculation for Falcon 9

In this section the code Example_11_03.m renamed as Example_Falcon.m is will be used to calculate the altitude where the stage separation begins.

The inputs are

- The Initial mass was calculated with the following data, being mass payload, mass structure for stage 1 and stage 2 and mass propellant for stage 1 and stage 2.

$$m_{payload} = 1862\ kg$$

$$m_{struture2} = 4900\ kg \quad m_{propellant}2 = 97000\ kg$$

$$m_{struture1} = 25000\ kg \quad m_{propellant1} = 414000\ kg$$

$$giving\ a\ total\ intial\ mass\ of\ \mathrm{m_0} = 542762\ \mathrm{Kg}$$

- To obtain the mass ratio for stage 1 the expected final mass will consider payload mass, total stage 2 mass and structure mass for stage 1. The next step is to consider an estimation of the mass propellant, thus from (Krause, 2017) an estimation of 55000 kg for propellant are consider for the reentry burn which will be consider in the final mass to obtain this parameter.

$$n = \frac{m_0}{m_f} = \frac{542762}{172762} = 3.14$$

- To obtain the thrust to weight ratio the thrust is considered to be 7607000 N. (SpaceX, 2020)

$$\mathrm{T2W} = \frac{\mathrm{Thrust}}{\mathrm{weight}} = \frac{7607000\ \mathrm{N}}{542762\ \mathrm{Kg}} = 1.4$$

- The specific impulse is Isp = 311 s (SpaceX, 2020)

- The Burnout mass is

$$mfinal = \frac{m0}{n}$$

- Rocket thrust, Thrust = 7607000 N

- Propellant mass flow

$$mdot = \frac{\frac{Thrust}{Isp}}{g0}$$

The results are:

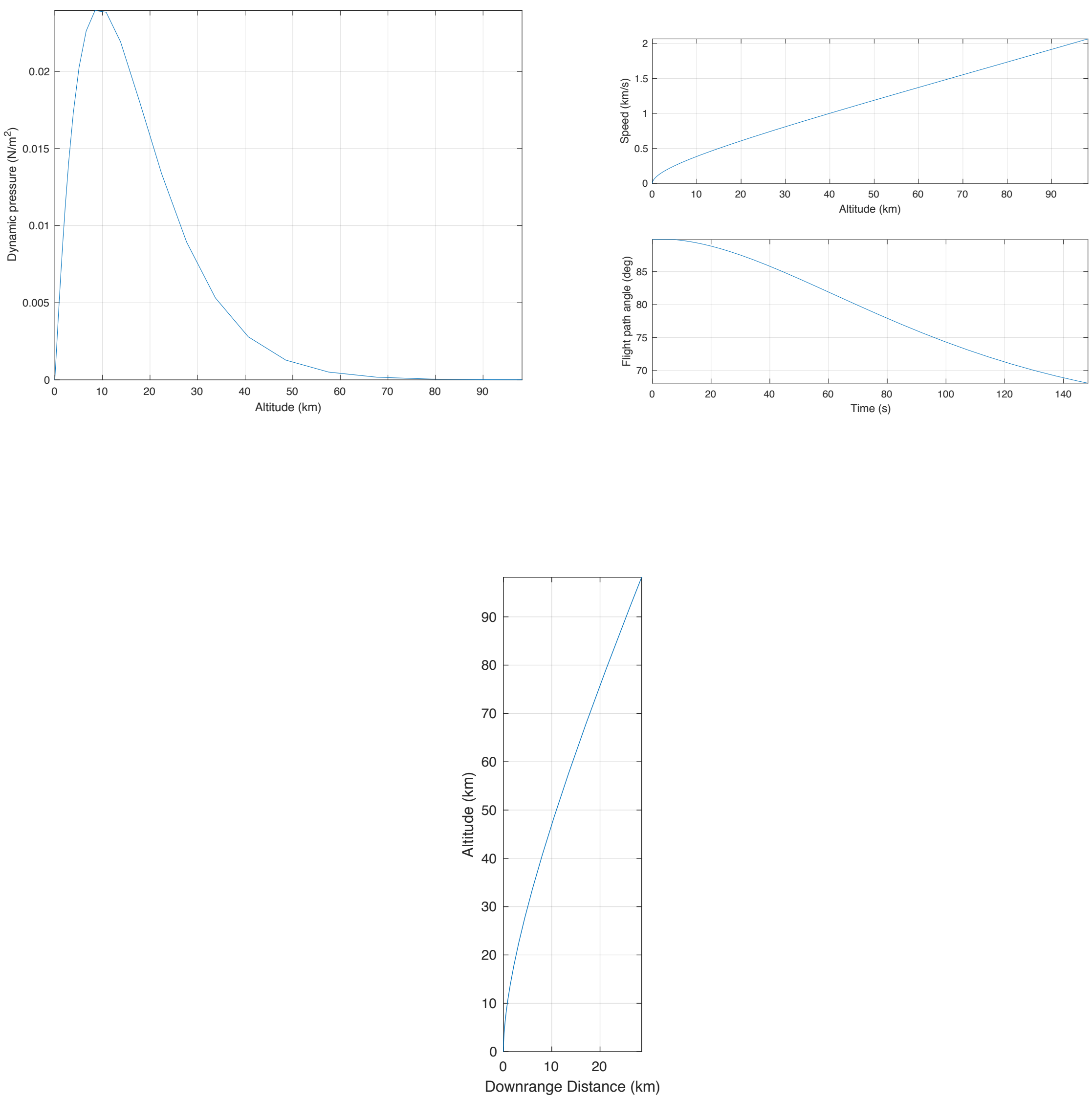


Figure 6. Validation of Altitude Calculation for Falcon 9 Iteration 1

- Initial flight path angle = 89.85 deg
- Pitch over altitude = 130 m
- Burn time = 148.358 s
- Final speed = 2.06543 km/s
- Final flight path angle = 68.0855 deg
- **Altitude = 98.1898 km**
- Downrange distance = 28.612 km
- Drag loss = 0.0182096 km/s
- Gravity loss = 1.40722 km/s

Now a second iteration of the altitude is presented for using 30% of the propellant mass according to previous research. The mass parameters have a change and are presented here.

The inputs are

- The Initial mass was calculated with the following data, being mass payload, mass structure for stage 1 and stage 2 and mass propellant for stage 1 and stage 2.

$$m_{payload} = 1862\ kg$$

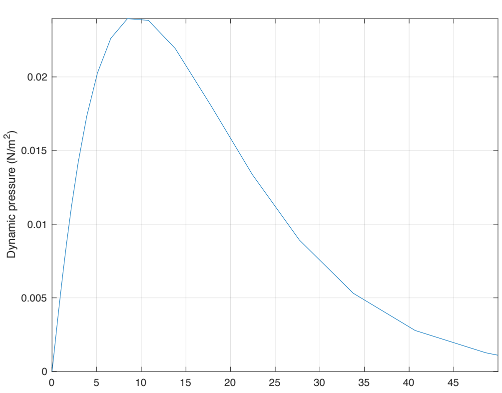
$$m_{struture2} = 4900\ kg \quad m_{propellant}2 = 97000\ kg$$

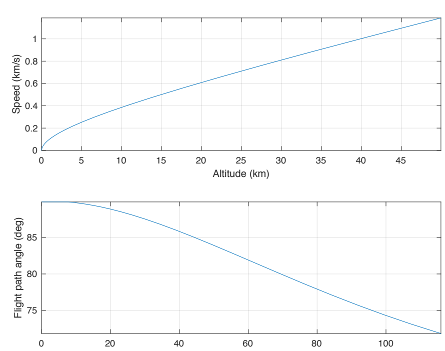
$$m_{struture1} = 25000\ kg \quad m_{propellant1} = 414000\ kg$$

$$giving\ a\ total\ intial\ mass\ of\ \mathrm{m}_0 = 542762\ \mathrm{Kg}$$

- To obtain the mass ratio for stage 1 the expected final mass will consider payload mass, total stage 2 mass and structure mass for stage 1. The next step is to consider an estimation of the mass propellant, of 30% which is 124200 kg, from the total propellant of stage1, this value is consider for the reentry burn which will be consider in the final mass to obtain this parameter.

$$n = \frac{m_0}{m_f} = \frac{542762}{252962} = 2.14$$





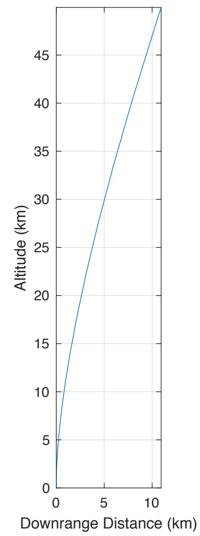
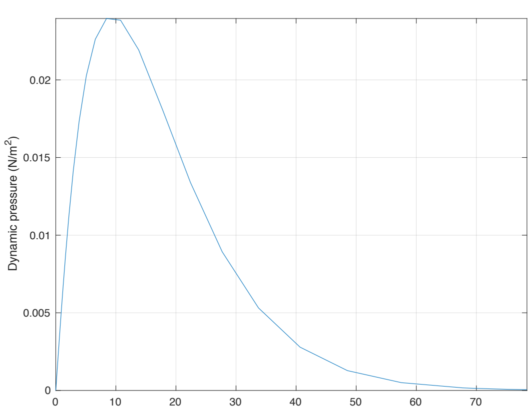
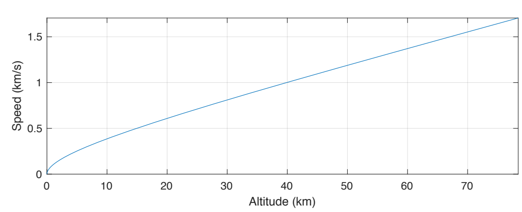


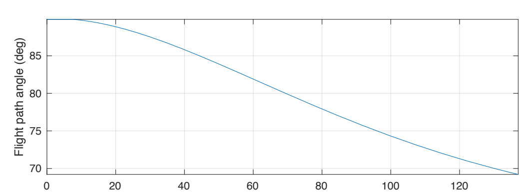
Figure 7.Figure 6. Validation of Altitude Calculation for Falcon 9 Iteration 2

- Initial flight path angle = 89.85 deg
- Pitch over altitude = 130 m
- Burn time = 115.962 s
- Final speed = 1.18754 km/s
- Final flight path angle = 71.8611 deg
- **<u>Altitude = 49.9537 km</u>**
- Downrange distance = 10.9407 km
- Drag loss = 0.0180325 km/s
- Gravity loss = 1.11556 km/s

Since both simulations gave two very different altitudes the following video (SpaceX, 2015) of the first return flight from Falcon 9 was reviewed and the altitude for stage separation was approximately 77 km. After several iterations this are the results for an altitude of 78 km with a mas ratio (n) of 2.7.







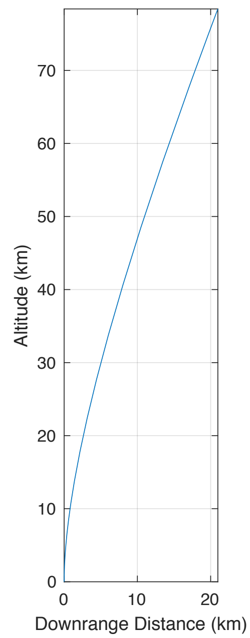


Figure 8. Figure 6. Validation of Altitude Calculation for Falcon 9 Iteration 3

- Initial flight path angle = 89.85 deg
- Pitch over altitude = 130 m
- Burn time = 137.06 s
- Final speed = 1.70534 km/s
- Final flight path angle = 69.2111 deg
- **Altitude = 78.4167 km**
- Downrange distance = 20.9825 km
- Drag loss = 0.0182035 km/s
- Gravity loss = 1.30678 km/s

After this validation we can use the value of n=2.7 with certainty and obtain the mass propellant for reentry in the following way.

$$m_f = \frac{m_0}{n} = \frac{542726}{2.7} = 201010\ kg$$

$$m_p = m_f - m_{strustureStage1} - m_{stage2} - m_{payload} = 201010 - 128726$$

$$m_{PropellantReentry} = 72284\ \mathrm{kg}$$

Which give us a 17.5% for mass propellant at reentry.

Calculation of Altitude for Prototype with 500 kg Payload

In this section the prototype design data is used as input to calculate the mass propellant for reentry

The inputs for the 500 kg prototype are

- The Initial mass was calculated with the following data, being mass payload, mass structure for stage 1 and stage 2 and mass propellant for stage 1 and stage 2.

$$m_{payload} = 500\ kg$$

$$m_{struture2} = 460.04\ \ kg \quad m_{propellant}2 = 3373.61\ kg$$

$$m_{struture1} = 3167.27\ kg \quad m_{propellant1LIFOFF} = 23226.6\ kg$$

$$m_{propellant1Reentry} = m_{propellant1LIFOFF} * 0.175 = 4064.5\ Kg$$

$$giving\ a\ total\ intial\ mass\ of\ \mathrm{m}_0 = 31133.5\ \ \mathrm{Kg}$$

- To obtain the mass ratio for stage 1 the expected final mass will consider payload mass, total stage 2 mass and structure mass for stage 1. The next step is to consider an estimation of the mass propellant, thus from (Krause, 2017) an estimation of 55000 kg for propellant are consider for the reentry burn which will be consider in the final mass to obtain this parameter.

$$m_f = \frac{m_0}{2.7} = \frac{31133.5}{2.7} = 11572\ kg$$

- The specific impulse is Isp = 311 s (SpaceX, 2020)

- The Burnout mass is

$$mfinal\ = \frac{m0}{n}$$

- Rocket thrust, Thrust = 654000 N

- Propellant mass flow

$$mdot = \frac{\frac{Thrust}{Isp}}{g0}$$

The results of the simulation are:

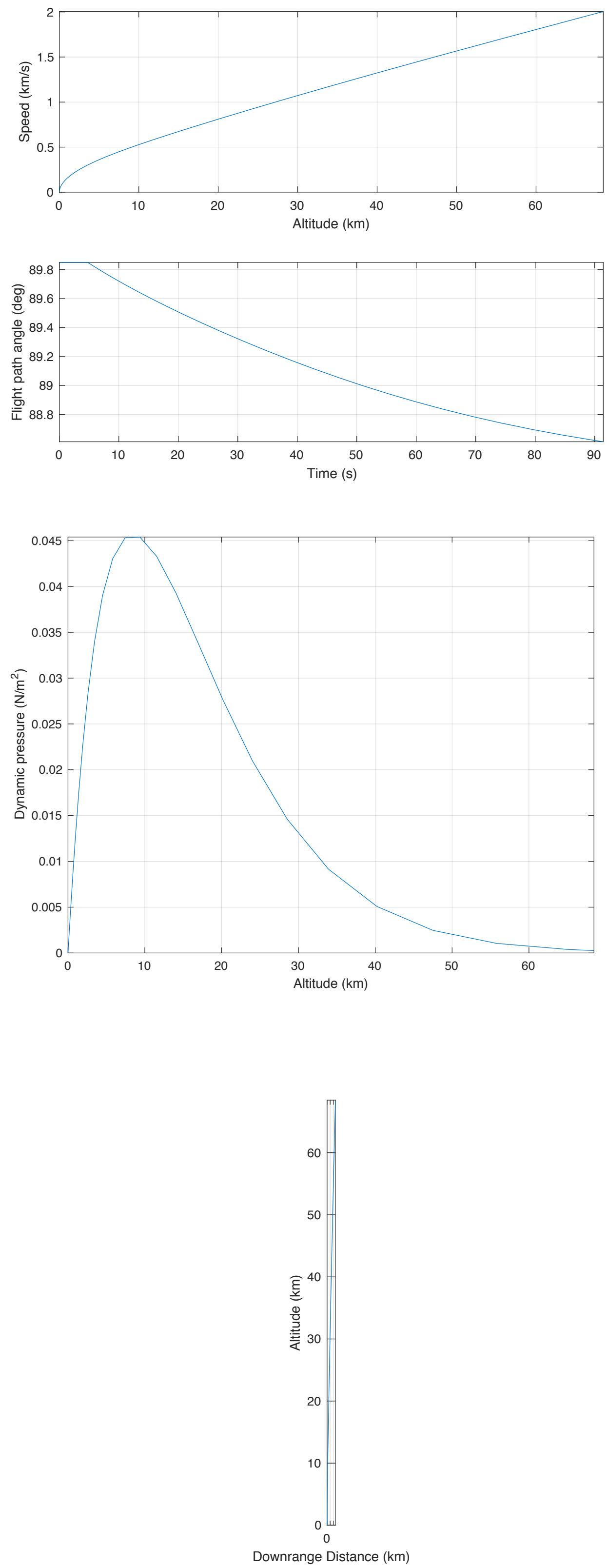


Figure 9. Calculation of Altitude for Prototype with 500 kg Payload

- Initial flight path angle = 89.85 deg
- Pitch over altitude = 130 m
- Burn time = 91.446 s
- Final speed = 2.00481 km/s
- Final flight path angle = 88.6118 deg
- Altitude = 68.4632 km
- Downrange distance = 1.33595 km
- Drag loss = 0.134075 km/s
- Gravity loss = 0.891388 km/s

Calculation of altitude for Prototype with 150 kg Payload

In this section the prototype design data is used as input to calculate the mass propellant for reentry

The inputs for the 150 kg prototype are

- The Initial mass was calculated with the following data, being mass payload, mass structure for stage 1 and stage 2 and mass propellant for stage 1 and stage 2.

$$m_{payload} = 150 \ kg$$

$$m_{struture2} = 138.01 \ kg$$
$$m_{propellant2} = 1012.08 \text{ kg}$$

$$m_{struture1} = 950.18 \ kg$$
$$m_{propellant1LIFOFF} = 7918.17 \ kg$$

$$m_{propellant1Reentry} = m_{propellant1LIFOFF} * 0.175 = 1385.77 \ Kg$$

Obtaining a total initial mass of

$$m_o = 10604.4 \text{ Kg}$$

- To obtain the mass ratio for stage 1 the expected final mass will consider payload mass, total stage 2 mass and structure mass for stage 1. The next step is to consider an estimation of the mass propellant, thus from (Krause, 2017) an estimation of 55000 kg for propellant are consider for the reentry burn which will be consider in the final mass to obtain this parameter.

$$m_f = \frac{m_0}{2.7} = \frac{10604.4}{2.7} = 13927.55 \ kg$$

- The specific impulse is Isp = 311 s (SpaceX, 2020)

- The Burnout mass is

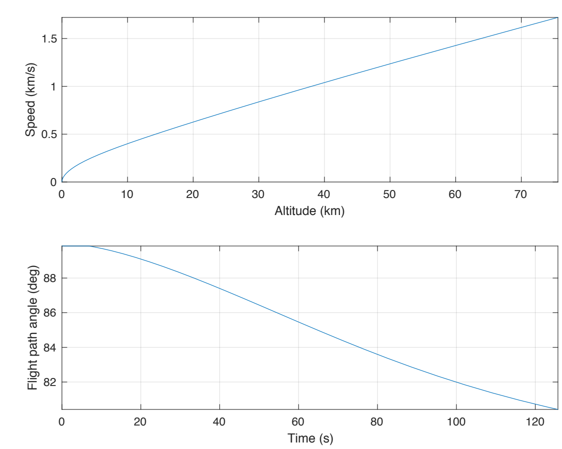
$$mfinal = \frac{m0}{n}$$

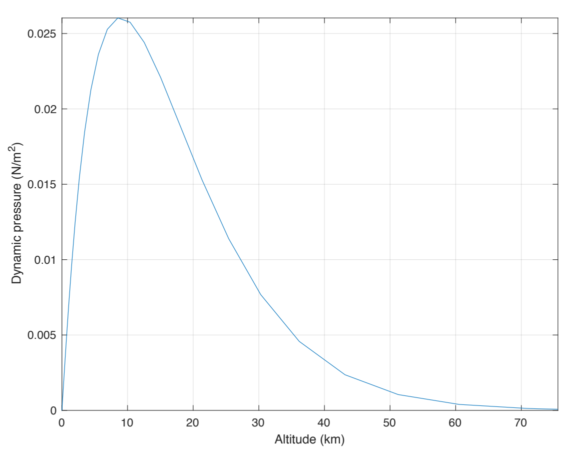
- Rocket thrust, thrust = 162,000 N, since the dimensions are very similar to Electron rocket the thrust selected is the same for this prototype. (Labs, 2019)

- Propellant mass flow

$$mdot = \frac{\frac{Thrust}{Isp}}{g0}$$

The results of the simulation are:





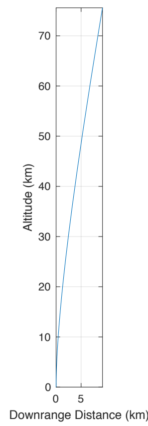


Figure 10. Calculation of altitude for Prototype with 150 kg

- Initial flight path angle = 89.85 deg
- Pitch over altitude = 130 m
- Burn time = 125.744 s
- Final speed = 1.72061 km/s
- Final flight path angle = 80.4186 deg
- <u>Altitude = 75.5806 km</u>
- Downrange distance = 9.27942 km
- Drag loss = 0.0902111 km/s
- Gravity loss = 1.21949 km/s

3.5 Calculation of descent acceleration

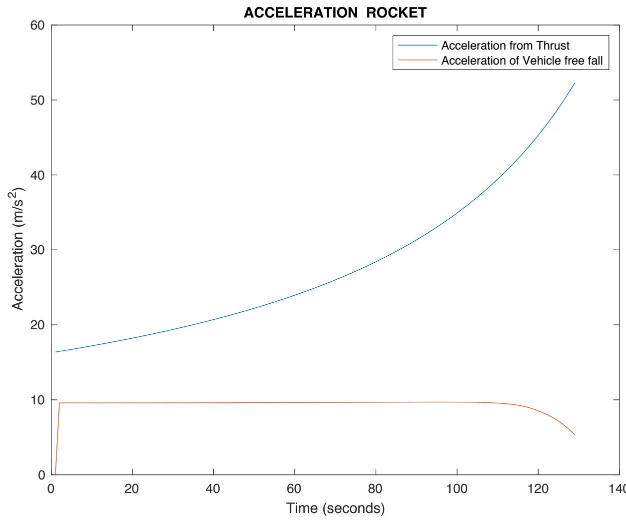
The calculation of descent is done through a MATLAB code called atmospheric_descent_F9.m (Walker, 2009) this code calculates the forces applied to the aircraft in the x, y and z position for gravity and drag, from this forces the velocity is calculated and then position of the aircraft at every second. In the code the function of acceleration in terms of the thrust and mass flow rate is incorporated. The results show this acceleration, the difference of acceleration at descent versus the one generated by the thrust and the altitude at each moment.

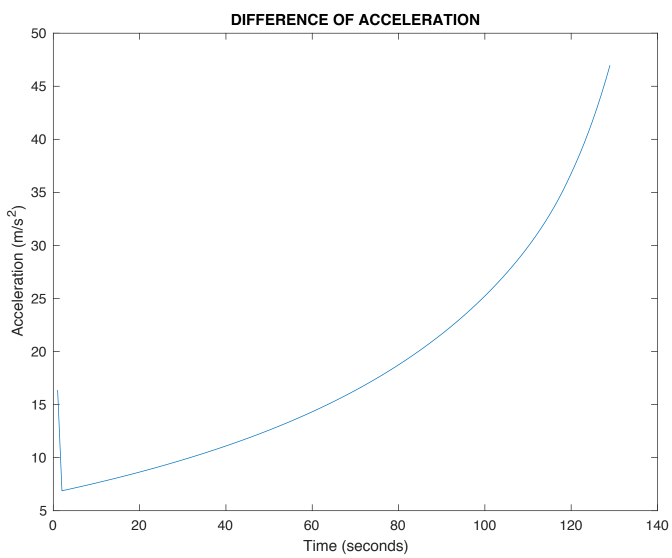
Validation for Falcon 9

The simulation uses the data of the Falcon 9 mission the altitude value is the one calculated previously. The key parameter here is the initial mass of reentry being the reentry propellant and the structure mass for the first stage.
The inputs are:

- Thrust= 7606000 N/ 2 engines
- Isp = 311 s
- Mass propellant = 72284 Kg
- Initial mass= 72284 Kg +25000 kg

- Starting Altitude for descent =78000 km





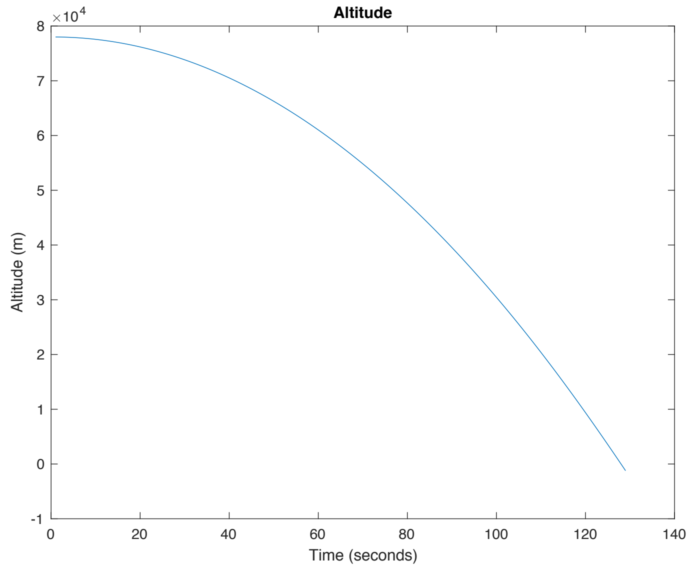


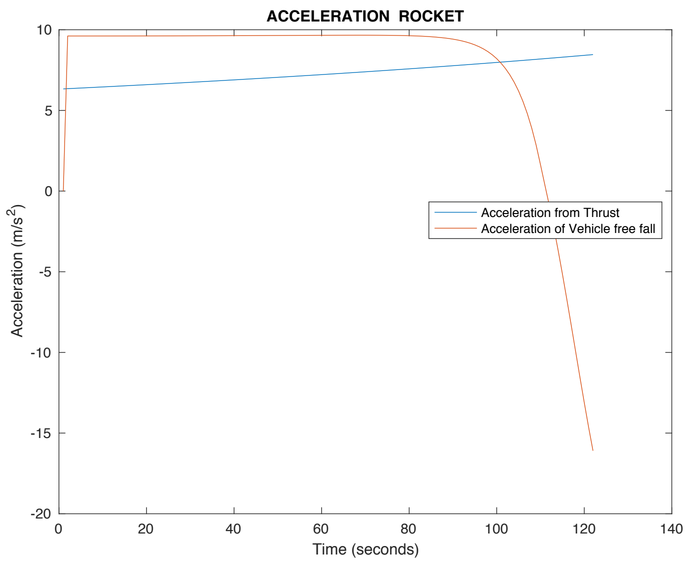
Figure 11Results of descent acceleration Falcon 9

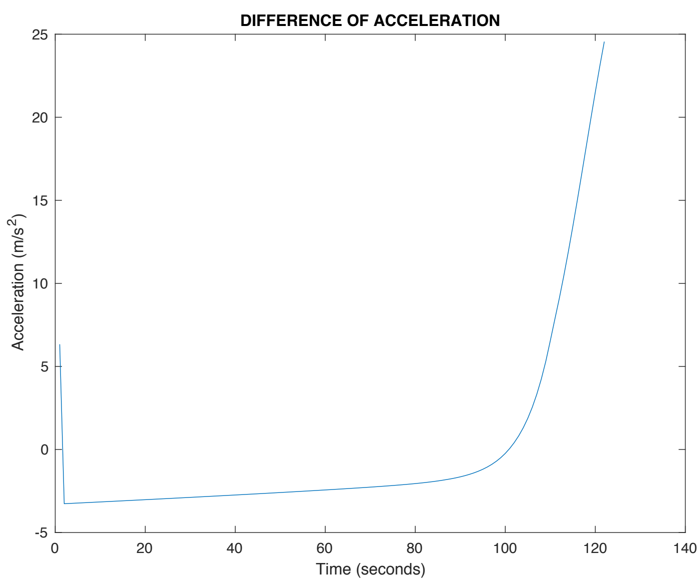
As we can see from the plots the first plot shows that the thrust overcomes the gravity acceleration of the reentry vehicle, the second graph is the difference between the two accelerations and by 120 seconds the vehicle lands, but in this simulation the thrust is larger than the acceleration descent this happens because as explained in previous sections the Falcon 9 turns on and off the busters and also changing the number of boosters from 3 to 1 depending on the altitude to have a save landing which is not consider in the simulation. Furthermore the approach is valid since it demonstrates that the mass propellant for reentry is more than enough to land, meaning the validation was successful.

Calculation of descent acceleration for prototype with a payload of 500 Kg

The simulation uses the data of the prototype for 500 Kg payload mission, the altitude value is the one calculated previously. The key parameter here is the initial mass of reentry being the reentry propellant and the structure mass for the first stage. The results show the acceleration in terms of time and the position of the vehicle through time until it lands

- Thrust= 654000 N
- Isp = 311 s
- Mass propellant = 9303.94 Kg
- Initial mass= 10254.1 kg
- Starting Altitude for descent =68000 km


ACCELERATION ROCKET
Acceleration (m/s2)
Time (seconds)
Acceleration from Thrust
Acceleration of Vehicle free fall
10
5
0
-5
-10
-15
-20
0
20
40
60
80
100
120
140


DIFFERENCE OF ACCELERATION
Acceleration (m/s2)
Time (seconds)
25
20
15
10
5
0
-5
0
20
40
60
80
100
120
140

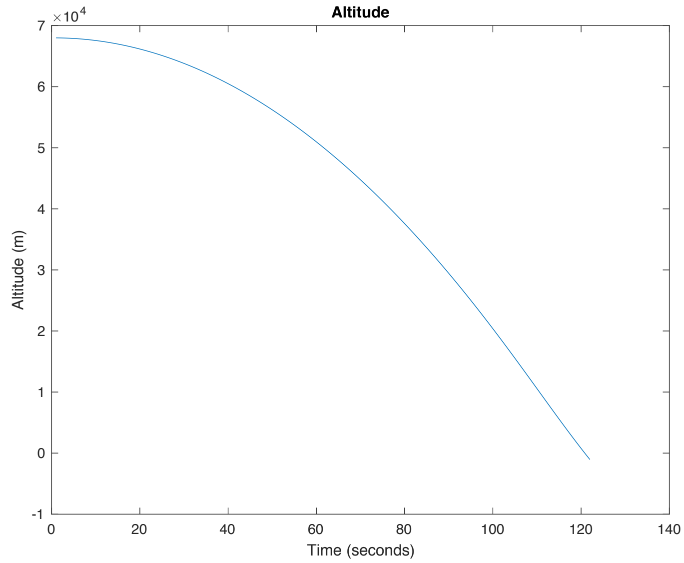


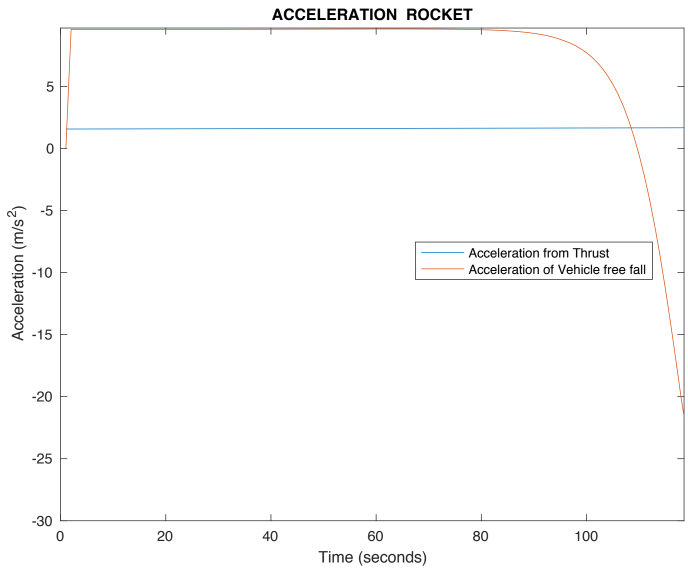
Figure 12. Results descent acceleration for prototype with a payload of 500 Kg

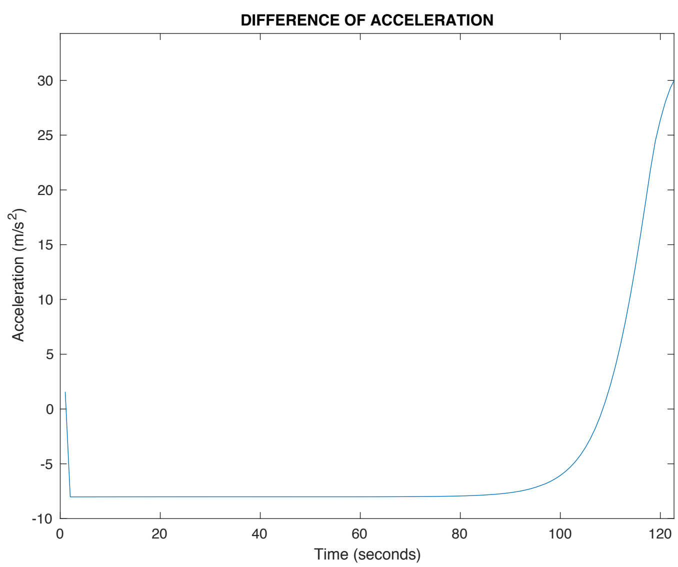
The results from the simulation in the first and second plot shows that the first 80 seconds of the reentry the acceleration is reduced but not canceled, after the 80 second the drag force is, and the thrust are higher than the deceleration. A factor that is not consider in this simulation are the hypersonic grid fins that help in the deceleration of the vehicle since the beginning of reentry and generate more drag force. Thus the reentry vehicle can achieve a landing with the mass propellant of reentry proposed and with the assistance of the grid fins and the deployable landing legs. The falcon 9 lands with a small amount of acceleration into the drone ship. The following results present a good approximation of how the prototype would behave in a real life scenario.

Calculation of descent acceleration for prototype with a payload of 150 Kg

The simulation uses the data of the prototype for 150 Kg payload mission, the altitude value is the one calculated previously. The key parameter here is the initial mass of reentry being the reentry propellant and the structure mass for the first stage

- Thrust= 162000 N
- Isp = 311 s
- Mass propellant = 1385.77Kg
- Initial mass= 2335.95 kg
- Starting Altitude for descent =75500 km


ACCELERATION ROCKET
Acceleration (m/s^2)
Time (seconds)
Acceleration from Thrust
Acceleration of Vehicle free fall
5
0
-5
-10
-15
-20
-25
-30
0
20
40
60
80
100


DIFFERENCE OF ACCELERATION
Acceleration (m/s^2)
Time (seconds)
30
25
20
15
10
5
0
-5
-10
0
20
40
60
80
100
120

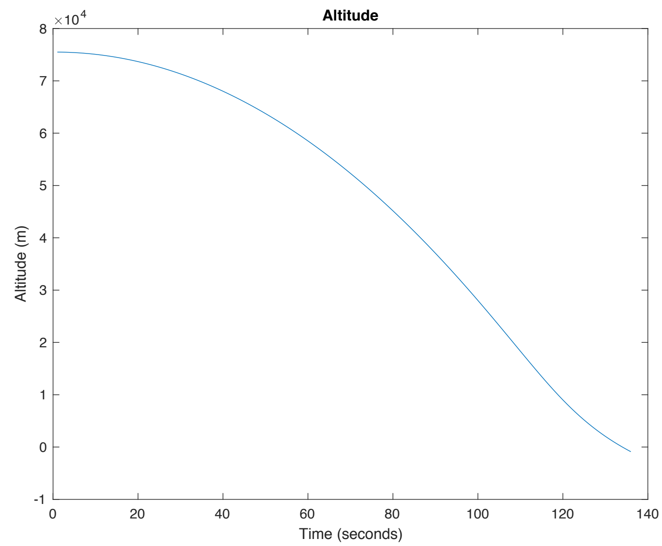
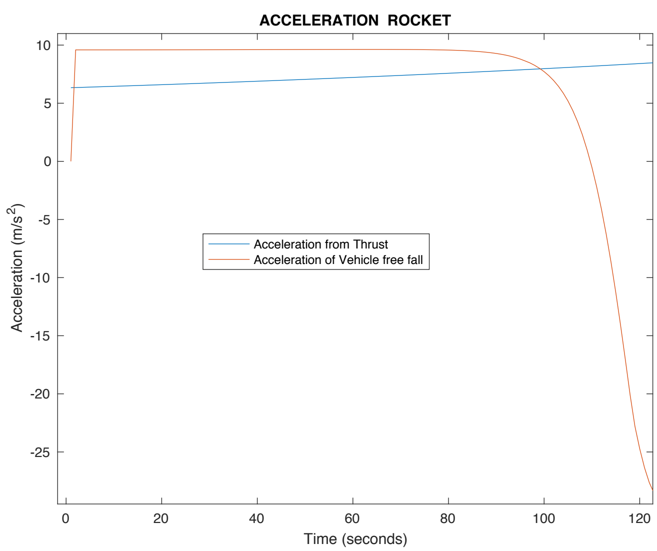
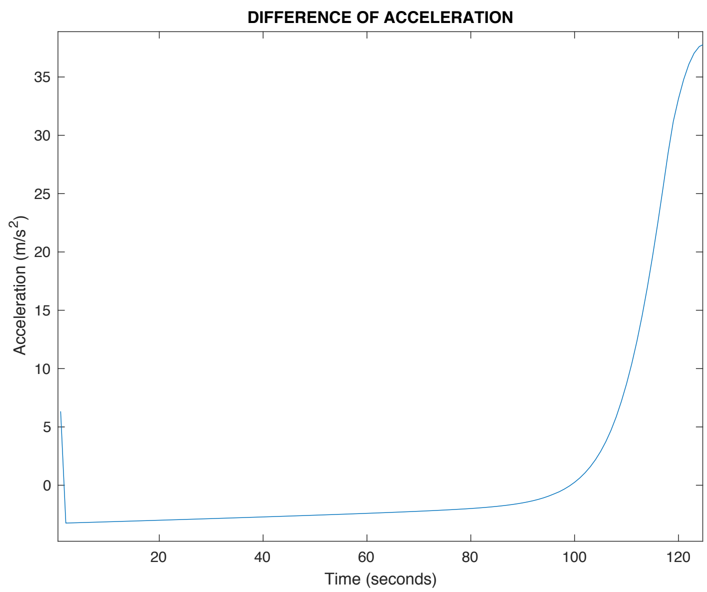


Figure 13 Results descent acceleration for prototype with a payload of 150 Kg

The results show that the Thrust generated by the characteristics of the rocket is not enough to decelerate the vehicle enough for a good landing. Even if the results show an increment in the deceleration due to drag at approximately 110 seconds the drag cannot be taken into the account since it will never generate a thrust force. Furthermore this configuration is not successful for landing even considering the assistance of aero breakers.

A second simulation was done using a thrust of 654000 N the results are presented.





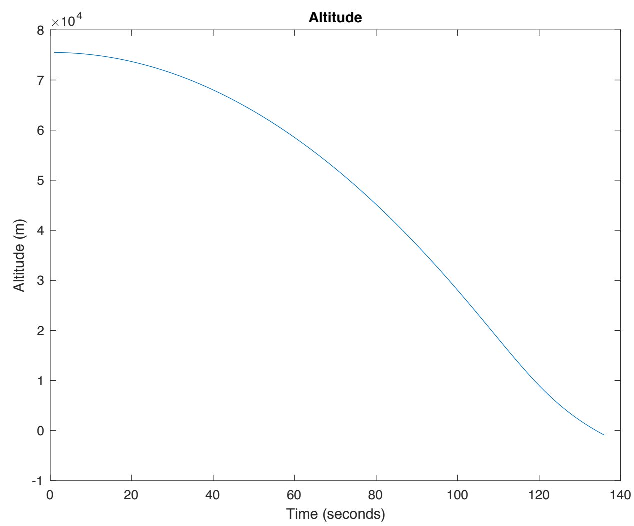


Figure 14 Second iteration Results descent acceleration for prototype with a payload of 150 Kg

The results for the final simulation show that with the increase of the thrust the acceleration at descent is reduced much better than the previous iteration. The set back of this configuration is that the amount of thrust is too high for a real configuration. Furthermore the approach can be optimized by developing a control law in the simulation that regulates the amount thrust in the vehicle while descending, this will optimize the amount of propellant used, as well as the structure mass. Finally a representation of the drag force generated by the hypersonic grid fins can be included to the simulation since they act as air brakers since the beginning of descent generating drag and thus this would reduce the amount of thrust and propellant needed for reentry.

3.6 Final estimation of masses and dimensions

Mass Estimation with reentry propellant

The following section presents the final results for the prototype in both configurations, 500 kg and 150 kg payload.

Parameter	Prototype 500kg	Prototype 150kg
Mass Payload	500Kg	150 Kg
Isp Stage 1	311 sec	311 sec
Isp Stage 2	345 sec	345 sec
Dv (Change of velocity to achieve 300 Km to Orbit)	9400 m/s	9400 m/s
TOTAL MASS FOR LIFTOFF	30727.56 kg	9218.26 kg
STAGE MASS 1	26393.9 kg	7918.17 kg
PROPELLANT MASS STAGE 1	23226.6 kg	6967.99 kg
STRUCTURE MASS STAGE 1	3167.27 kg	950.18 kg
STAGE MASS 2	3833.65 kg	1300.09 kg
PROPELLANT MAS STAGE 2	3373.61 kg	1012.08 kg
STRUCTURE MASS STAGE 2	460.04 kg	138.01 kg
PROPELLANT MASS FOR REENTRY	4064.5 kg	1385.77 kg
TOTAL MASS LIFTOFF AND REENTRY	35332.1 kg	10604 kg

Parameter	Prototype 500kg	Prototype 150kg
SIGMA STAGE 1	**0.2667**	**0.2667**
Mass Payload	500Kg	150 Kg
Isp	311 sec	311 SEC
Isp Stage 2	345 sec	345 sec
Dv (Change of velocity to achieve 500 Km SUN-SYNC. orbit)	9400 m/s	9400 m/s
TOTAL MASS	95846.72	28754.016
STAGE MASS 1	83737.0012	25121.1
PROPELLANT MASS STAGE 1	61404.3	18421.20
STRUCTURE MASS STAGE 1	22332.65	6699.77
STAGE MASS 2	11609.71	3482.91
PROPELLANT MAS STAGE 2	10216.32	3064.96
STRUCTURE MASS STAGE 2	1393.12	417.94

The results show a reduction form the first iteration of mas calculation by almost 30%. This demonstrates that the optimization and iteration in the design projects are key to achieve the best result. Specially in a reusability system where it has to be optimized in order to be viable, but the inconvenience comes from the propulsion systems that has multiple variables and is affected at the same time by a complex environment which is hard to simulate.

Dimensions 500 kg payload

Table with optimization propellant reentry

STAGE 1	STAGE 2
THRUST= 654KN	**THRUST 22KN**
Throat Area=0.001087	Throat Area=0.001096
Throat diameter =0.037206	Throat diameter =0.037354
Nozzle exit area =0.017396	Nozzle exit area =0.017534
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.149415
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.167288
Thrust chamber volume=0.001852	Thrust chamber volume=0.001867
Thrust chamber area=0.004349	Thrust chamber area=0.004383
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.074708
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =16.659754	Oxidizer tank volume =1.748729
Oxidizer tank diameter=2.000000	Oxidizer tank diameter=2.000000
Fuel tank volume =10.253954	Fuel tank volume =1.076330
Fuel tank diameter =2.000000	Fuel tank diameter =2.000000
Oxidizer tank lenght =3.459771	Oxidizer tank lenght =0.363163
Fuel tank lenght =5.621142	Fuel tank lenght =0.590036
TOTAL LENGHT=10.316029	TOTAL LENGHT=1.888976

Table without optimization of propellant reentry

STAGE 1	STAGE 2
Throat Area=0.001087	Throat Area=0.001096
Throat diameter =0.037206	Throat diameter =0.037354
Nozzle exit area =0.017396	Nozzle exit area =0.017534
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.149415
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.167288
Thrust chamber volume=0.001852	Thrust chamber volume=0.001867
Thrust chamber area=0.004349	Thrust chamber area=0.004383
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.074708
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =37.484859	Oxidizer tank volume =6.236653
Oxidizer tank diameter=2.000000	Oxidizer tank diameter=2.000000
Fuel tank volume =23.071650	Fuel tank volume =3.838613
Fuel tank diameter =2.000000	Fuel tank diameter =2.000000
Oxidizer tank lenght =7.784570	Oxidizer tank lenght =1.295181
Fuel tank lenght =12.647709	Fuel tank lenght =2.104300
TOTAL LENGHT=21.667396	TOTAL LENGHT=4.335258

The results in the table present a drastic reduction from the first iteration of reusable version. The optimization helped reduced the length by almost half in both stages. The selection of the diameter for this configuration was for two meters due to the fact that the Electron has a diameter of 1.2 meters and Falcon 9 has a 3.6 diameter this prototype rocket was in between does a 2 meter diameter was a fitted option for design.

Dimensions 150 kg payload

Table with optimization propellant reentry.

STAGE 1	**STAGE 2**
THRUST= 654KN	**THRUST 22KN**
Throat Area=0.001087	Throat Area=0.001096
Throat diameter =0.037206	Throat diameter =0.037354
Nozzle exit area =0.017396	Nozzle exit area =0.017534
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.149415
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.167288
Thrust chamber volume=0.001852	Thrust chamber volume=0.001867
Thrust chamber area=0.004349	Thrust chamber area=0.004383
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.074708
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =5.099635	Oxidizer tank volume =0.617834
Oxidizer tank diameter=1.100000	Oxidizer tank diameter=1.100000
Fuel tank volume =3.138787	Fuel tank volume =0.380272
Fuel tank diameter =1.100000	Fuel tank diameter =1.100000
Oxidizer tank lenght =3.501003	Oxidizer tank lenght =0.424156
Fuel tank lenght =5.688133	Fuel tank lenght =0.689132
TOTAL LENGHT=10.424251	TOTAL LENGHT=2.049065

Table without optimization of propellant reentry

STAGE 1	**STAGE 2**
Throat Area=0.001087	Throat Area=0.001096
Throat diameter =0.037206	Throat diameter =0.037354
Nozzle exit area =0.017396	Nozzle exit area =0.017534
Nozzle exit diameter=0.148824	Nozzle exit diameter=0.149415
Nozzle exit lenght=0.166626	Nozzle exit lenght=0.167288
Thrust chamber volume=0.001852	Thrust chamber volume=0.001867
Thrust chamber area=0.004349	Thrust chamber area=0.004383
Thrust chamber diameter=0.074412	Thrust chamber diameter=0.074708
Thrust chamber lenght=0.468490	Thrust chamber lenght=0.468490
Oxidizer tank volume =11.245403	Oxidizer tank volume =1.871035
Oxidizer tank diameter=1.100000	Oxidizer tank diameter=1.100000
Fuel tank volume =6.921461	Fuel tank volume =1.151608
Fuel tank diameter =1.100000	Fuel tank diameter =1.100000

Oxidizer tank lenght =7.720197	Oxidizer tank lenght =1.284503
Fuel tank lenght =12.543122	Fuel tank lenght =2.086952
TOTAL LENGHT=21.498435	TOTAL LENGHT=4.307233

The results in this case demonstrate a reduction of the length by half. For this specific prototype the results were not completely viable since the first stage had to have a very large thrust to attempt a recovery, it still demonstrates that the optimization and iteration is key to solve this type of systems. A better fit for this prototype would be developing a control program for the thrust at descent to optimize the reentry. This would change the amount of propellant an does would give a more exact approximation of the vehicle proposed.

Conclusions

The new space market every day is developing smaller and more powerful technology. This means that the launcher sector needs to be capable of deliver options for this new market. This thesis project presents an option for small payloads using a reusable launcher vehicle. The reusable launcher systems have a variety of techniques to recover a part or almost all the vehicle. Most of the recovery techniques are usually a constraint for the vehicle or have a high price value such that it is not profitable even for a governmental institution. Thus the reentry boost system together with the hypersonic grid fins and landing gear legs of the rocket Falcon 9 developed by Space X, are the most optimal option to research. Also it is very important to mention that the design of the Falcon 9 is very optimized, thus because of this efficient propulsion system Space X has obtained multiple landings successfully and by consequence obtaining more funding and missions. The research shows more and more companies are interested in incorporating this technology to the space market, since it has been proved to be possible. For this reason the recovery technique and system selected to focus on was the reentry propulsion system with the extra subsystems.

The first iteration of design for a rocket vehicle including a reusability systems was done by calculation of basic propulsion theory. This calculations demonstrated that the propulsion system is highly constrained by the specific impulse of the engine, the propellant characteristics, chamber pressure and mass ratios. Then values such as mass flow rate, thrust and dimensions of engine, propellant tank can be calculated. Because of the first approximation the value of the thrust for the prototype was calculated with a value of 654 kN and a method to obtain the dimension of the rocket was also developed during the first iteration where the results were a rocket with a mass of approximately 25 tons with a length of 24 meters but only for a 300km LEO mission. Then the idea of developing two different prototypes was presented with two different payload masses. The following section explains the important of the smaller subsystems of the reusable vehicle, which are landing legs and hypersonic grid fins Also a first estimation of the reusability mass propellant was developed based on a previous mission of the Falcon 9, where the results estimated that 13% of the mass propellant of the first stage was used for reentry, but other data from Space X company suggested the this value was for 30%. Furthermore after reviewing the data with real values the decision was made to use a stagging mass method to obtain a more optimized value of the stages. This method was validated and gave optimal results for the mass of the rocket such as approximately 9 tons of mass for the prototype at 150 kg and 30 tons for the prototype with payload of 500 kg, this data was only done for liftoff. For the next iteration the reusability system had to be included with a 30% increment of mass the values went up very high for the prototype of 500 kg payload the mass was 90 tons with a diameter of 2 meters and almost 30 m of length , and for the prototype of 150kg payload it went up to more than 90 tons a diameter of 1.1m with 24 m of length. Since the exact number of the amount of propellant needed was not obtained due to the fact that very few information is available on this reusable system, most of the propulsion books do not cover this topic and the fact that is a new technology in terms that a real vehicle has been developed less than 5 years ago a different approach od estimation of this mass propellant was implemented. With the original optimized values of the rocket only for liftoff the maximum altitude for the first stage was calculated for Falcon 9 this gave a mass ratio n=2.7 with this value the percentage of propellant mass for reentry was obtained and applied for both prototypes. Continuing with the a descent reentry simulation for both cases demonstrated that the fuel was able to decelerate the returning first stage of the 500 kg payload and for the case of 150kg payload it was not able to recover with that amount of propellant unless the thrust was increased thus this change may affect the total design of the rocket thus is not successful. The final dimension for the 500 kg payload prototype were the mass for 35.6 tons with a diameter of 2 meters and a length of 12meters and for the 150 kg payload prototype its mass for 10.6 tons a diameter of 1.1 meters and a length of 13 meters.

Finally the prototype presented for 500 kg payload demonstrated to be a feasible design since it can carry the payload to one of the most common orbits for small satellites and is able to recover the first stage. The second prototype for 150kg was not able to fully recover the first stage since the deceleration was not enough, apart from this results both of these reusable launcher vehicles with this characteristics may improve the price market of the launching systems. Furthermore the continuous design and optimization of this vehicle should be done through control systems evaluating the propellant mass according to the reentry conditions once this technology is perfected a large reduction in the structure mass and in the total mass of the rocket will be achieved obtaining a larger economic impact for the company developing this type of vehicle. Even though the design and testing of such systems can be expensive once the rocket is fully functional the profit will come once the vehicle is used in multiple trips and at the same time there will be a cost reduction in the space launching market.

Future work

- The first recommendation for future work is to have a CFD software to calculate the engines performance as well as dimensioning the tanks, even though the codes and the propulsion theory are up to date there are still a lot of values that have to be assumed by previous projects. This calculations can be more exact with the welp of a Computational Fluid Dynamics software. Due to the fact that this thesis project was done during a pandemic and most of the time the school facilities were closed there was no access to such software's thus becoming difficult to go for this route.

- Its recommended to study further the impact of the subsystems of the recovery systems such as the landing gear legs and the hypersonic grid fins. For the legs is recommended to scale them down to the rockets dimensions and use a computer aided design software to estimate the loads that the landing legs may experience. For the hypersonic grid fins a scale down design is recommended and a study of how this grid fins decelerate the vehicle through the reentry mainly the drag effect.

-Another recommendations is to develop a control system for the thrust at reentry this will assist in getting the exact amount of propellant during the reentry. The control system should obtain the information by the altitude and speed of the rocket during the descent. This will optimize the design of the rocket at liftoff and have a more precise design for this types of vehicles.

-The final recommendation is to take into account the design of the length with the diameter, and the mass of the structure shell for the whole body and mass of the extra subsystems needed for recovery and the drome ships where usually this type of rockets land. Furthermore a combination of reentry burn and another type of recovery system is recommended to investigate its viability.

MATLAB CODE

MATLAB Code

This section presents the codes used for the results obtained.

STAGE_1_DESIGN.m

```
%% STAGE 1 DESIGN

clear all
clc
%   CHAMBER DESIGN PARAMETERS %
syms w m real;
m_p2= 3260       ;% Kg     MASS STAGE 2
D_v= 4300      ;%    m/s
F_v= 654000   ;%    Newtons at SEA LEVEL
Isp2= 282  ; % s @ vaccum
```

```
g=9.806  ;   %m/s^2
r= 2.3
c_star= 1820    % cstar characteristic velocity table 5-5 RP1
p1=  10.8*10^6  %Pa    chamber pressu^re manual Rocket
df=806          %   Density fuel
do=1141          %   Density oxidizer
ee= 16      % selected nozzle area ratio      AREA RATIO FROM AUSTRONAUTIX
da= 1014     % average density
k=1.24

R=8314.3/23% Gas constant
Dtank=1.1    % assumption of 1.2 meter for design in first stage and previous
rocket
ts=1*10^-3
Li= 0.60   % m  injectors and valve feed system
p3=  0.101325*10^6      %Atmospheric pressure

% CALCULATIONS %

mP= m_p2*(exp(D_v/(Isp2*g))-1)/(1.07-0.07*exp(D_v/(Isp2*g)));  %propellant
mass
m_st= 0.07*mP       %strucure mass
m_T2=mP+m_st+m_p2        %total mass
m_stage=mP+m_st
mdot=F_v/(Isp2*g)       % mass flow rate

mdotf=mdot/(r+1)        % mass flow rate fuel
mdot0=mdot*r/(r+1)      % mass flow gas

mdotfTC=mdotf*0.98      % mass flow rate fuel in thrust chamber
mdot0TC=mdot0*0.98      % mass flow rate gas in thrust chamber
mdotfgg= 0.02*mdotf;
mdot0gg= 0.02*mdot0;


mdotp= mdot0TC-  mdotfTC    %    mass flow rate propellant

tb=mP/mdot          %  Duration

   % Thrust coefficioent
n=((k+1)/(2*(k-1)))
M2=vpasolve(ee==(1/m)*((2+(k-1)*(m^2))/(k+1))^n ,m)

p2=p1/(1+((k-1)/2)*M2^2)^(k/(k-1))
pt=p1*((2/(k+1))^(k/(k-1)))

C_F=((((2*k^2)/(k-1))*((2/(k+1))^((k+1)/(k-1)))*((1-(p2/p1)^((k-
1)/k))))^(1/2))+((p2-p3)/p1)*ee
Isp222=(C_F*c_star)/g

At=F_v/(C_F*p1)       % Area throat
A2=ee*At              % Area exit area nozzle
T2= (k/R)*(((p2*A2*M2)/mdot)^2)
T1=T2*(1+((k-1)/2)*M2^2)
Tt=(2*T1)/(k+1)

dt=sqrt(4*At/pi)       % Diameter at throat
```

```
d2=dt*sqrt(ee)        % Diameter exit



r_v=r*(df/do)
vt=mP/da               %Total volume

vtp=vt/(r_v+1)       % volume  fuel
vto=(vt*r_v)/(r_v+1)   % volume  oxydizer

Ltp= (vtp)/(pi*((Dtank/2)^2))
Lto=(vto)/(pi*((Dtank/2)^2))

Ltpp=Ltp*1.06      %Lenght tank fuel
Ltop=Lto*1.06      % Lenght tank oxydizer


L2=0.8*((d2-dt)/(2*tand(15)))     % Chamber temperature

L_c=0.4259 %m From dt and Previous chamber test
dc=2*dt
Ac=pi*(dc/2)^2
VC= L_c*Ac      %area of the chamber

%L_c=1.1*VC2/Ac       % Lenght of the chamber
L_c2=1.1*VC/Ac

LT=L2+Ltop+Ltpp+L_c+Li

%t111=vpasolve(VC2==mdot*ts*R*TT/p1)
%T22= (k/R)*((p2*A2*M2)/mdot)^2

fprintf('Mass propellant= %.2f \n',mP)
fprintf('Burning time = %f\n',tb)
fprintf('Total mass flow = %f\n',mdot) ;
fprintf('Mass flow Oxidizer in gas generator = %f\n',mdot0gg)
fprintf('Mass flow Fuel in gas generator= %f\n', mdotfgg)
fprintf('Mass flow Oxidizer in thrust chamber = %f\n' , mdot0TC)
fprintf('Mass flow Fuel in thrust chamber=%f\n' , mdotfTC)
fprintf('Exit Mach =%f \n' , M2)
fprintf('Exit pressure =%f \n' , p2)
fprintf('Throat pressure =%f\n' , pt)
fprintf('Chamber temperature =%f\n', T1)
fprintf('Throat temperature =%f\n',Tt)
fprintf('Exit temperature =%f\n',T2)
fprintf('Throat Area=%f\n' , At)
fprintf('Throat dimater =%f\n', dt)
fprintf('Nozzle exit area =%f\n', A2)
fprintf('Nozzle exit diameter=%f\n',d2)
fprintf('Nozzle exit lenght=%f\n' , L2)
fprintf('Thrust chamber volume=%f\n', VC )
fprintf('Thrust chamber area=%f\n' ,Ac)
fprintf('Thrust chamber diameter=%f\n',dc)
fprintf('Thrust chamber lenght=%f\n' ,L_c)
fprintf('Oxidizer tank volume =%f\n' , vto)
fprintf('Oxidizer tank diamater=%f\n',Dtank)
fprintf('Fuel tank volume =%f\n',vtp)
```

```
fprintf('Fuel tank diamater =%f\n', Dtank)
fprintf('Oxidizer tank lenght =%f\n', Ltpp)
fprintf('Fuel tank lenght =%f\n', Ltop)
fprintf('TOTAL LENGHT=%f\n',LT)
fprintf('TOTAL MASS STAGE 1 =%f\n',m_stage)
fprintf('TOTAL MASS =%f\n',m_T2)
fprintf('STRUCTURE MASS STAGE 1 =%f\n', m_st)
fprintf('PROPELLANT MASS STAGE 1 =%f\n', mP)
```

STAGE_2_Design.m

```
clear all
clc
%   CHAMBER DESIGN PARAMETERS %
syms w m real;
m_p= 500       ;% Kg     payload mass
D_v= 5300      ;%    m/s
F_v= 981000/5   ;%    Newtons
Isp2= 345  ; % s @ vaccum
g=9.806  ;   %m/s^2
r= 2.3
c_star= 1820    % cstar characteristic velocity table 5-5 RP1
p1=  10.8*10^6  %Pa    chamber pressu^re manual Rocket
df=806          %   Density fuel
do=1141          %   Density oxidizer
ee= 16      % selected nozzle area ratio      AREA RATIO FROM AUSTRONAUTIX
da= 1014     % average density
k=1.23

R=8314.3/23% Gas constant
Dtank=1.1    % assumption of 1.2 meter for design in first stage and previous
rocket
ts=1*10^-3
Li= 0.30   % m  injectors and valve feed system

% CALCULATIONS %

mP= m_p*(exp(D_v/(Isp2*g))-1)/(1.07-0.07*exp(D_v/(Isp2*g)));  %propellant
mass
m_st= 0.07*mP       %strucure mass
m_T2=mP+m_st+m_p        %total mass
m_stage=mP+m_st
mdot=F_v/(Isp2*g)       % mass flow rate

mdotf=mdot/(r+1)        % mass flow rate fuel
mdot0=mdot*r/(r+1)      % mass flow gas

mdotfTC=mdotf*0.98      % mass flow rate fuel in thrust chamber
mdot0TC=mdot0*0.98      % mass flow rate gas in thrust chamber
mdotfgg= 0.02*mdotf;
mdot0gg= 0.02*mdot0;

mdotp= mdot0TC-  mdotfTC    %    mass flow rate propellant

tb=mP/mdot          %  Duration
```

```
C_F=Isp2*g/c_star    % Thrust coefficioent

At=F_v/(C_F*p1)       % Area throat
A2=ee*At              % Area exit area nozzle

dt=sqrt(4*At/pi)       % Diameter at throat
d2=dt*sqrt(ee)        % Diameter exit

n=((k+1)/(2*(k-1)))
M2=vpasolve(ee==(1/m)*((2+(k-1)*(m^2))/(k+1))^n ,m)

r_v=r*(df/do)
vt=mP/da               %Total volume

vtp=vt/(r_v+1)       % volume  fuel
vto=(vt*r_v)/(r_v+1)   % volume  oxydizer

Ltp= (vtp)/(pi*((Dtank/2)^2))
Lto=(vto)/(pi*((Dtank/2)^2))

Ltpp=Ltp*1.06      %Lenght tank fuel
Ltop=Lto*1.06      % Lenght tank oxydizer


p2=p1/(1+((k-1)/2)*M2^2)^(k/(k-1))
T2= (k/R)*(((p2*A2*M2)/mdot)^2)

T1=T2*(1+((k-1)/2)*M2^2)

pt=p1*((2/(k+1))^(k/(k-1)))
Tt=(2*T1)/(k+1)

L2=0.8*((d2-dt)/(2*tand(15)))     %  nozzle lenght

L_c=0.4259 %  From dt and Previous chamber test
dc=2*dt
Ac=pi*(dc/2)^2
VC= L_c*Ac      %area of the chamber

%L_c=1.1*VC2/Ac       % Lenght of the chamber
L_c2=1.1*VC/Ac

LT=L2+Ltop+Ltpp+L_c+Li

%t111=vpasolve(VC2==mdot*ts*R*TT/p1)
%T22= (k/R)*((p2*A2*M2)/mdot)^2

fprintf('Mass propellant= %.2f \n',mP)
fprintf('Burning time = %f\n',tb)
fprintf('Total mass flow = %f\n',mdot) ;
fprintf('Mass flow Oxidizer in gas generator = %f\n',mdot0gg)
fprintf('Mass flow Fuel in gas generator= %f\n', mdotfgg)
fprintf('Mass flow Oxidizer in thrust chamber = %f\n' , mdot0TC)
fprintf('Mass flow Fuel in thrust chamber=%f\n' , mdotfTC)
fprintf('Exit Mach =%f \n' , M2)
fprintf('Exit pressure =%f \n' , p2)
```

```
fprintf('Throat pressure =%f\n' , pt)
fprintf('Chamber temperature =%f\n', T1)
fprintf('Throat temperature =%f\n',Tt)
fprintf('Exit temperature =%f\n',T2)
fprintf('Throat Area=%f\n' , At)
fprintf('Throat dimater =%f\n', dt)
fprintf('Nozzle exit area =%f\n', A2)
fprintf('Nozzle exit diameter=%f\n',d2)
fprintf('Nozzle exit lenght=%f\n' , L2)
fprintf('Thrust chamber volume=%f\n', VC )
fprintf('Thrust chamber area=%f\n' ,Ac)
fprintf('Thrust chamber diameter=%f\n',dc)
fprintf('Thrust chamber lenght=%f\n' ,L_c)
fprintf('Oxidizer tank volume =%f\n' , vto)
fprintf('Oxidizer tank diamater=%f\n',Dtank)
fprintf('Fuel tank volume =%f\n',vtp)
fprintf('Fuel tank diamater =%f\n', Dtank)
fprintf('Oxidizer tank lenght =%f\n', Ltpp)
fprintf('Fuel tank lenght =%f\n', Ltop)
fprintf('TOTAL LENGHT=%f\n',LT)
fprintf('TOTAL MASS STAGE 2 =%f\n',m_stage)
fprintf('TOTAL MASS =%f\n',m_T2)
fprintf('STRUCTURE MASS STAGE 2 =%f\n', m_st)
fprintf('PROPELLANT MASS STAGE 2 =%f\n', mP)
```

OptiProtoStage1Tanks.m

```
clear all
clc
%   CHAMBER DESIGN PARAMETERS %
syms w m real;
m_p2= 3482.91+150     ;% Kg     MASS STAGE 2 PLUS PAYLOAD
D_v= 4300      ;%    m/s
F_v= 19000   ;%    Newtons at SEA LEVEL  from Electron rocket
Isp2= 311  ; % s @ vaccum
g=9.806  ;   %m/s^2
r= 2.3
c_star= 1820    % cstar characteristic velocity table 5-5 RP1
p1=  10.8*10^6  %Pa    chamber pressu^re manual Rocket
df=806          %   Density fuel
do=1141          %   Density oxidizer
ee= 16      % selected nozzle area ratio      AREA RATIO FROM AUSTRONAUTIX
da= 1014     % average density
k=1.24

R=8314.3/23% Gas constant
Dtank=1.1    % assumption of 1.2 meter for design in first stage and previous
rocket
ts=1*10^-3
Li= 0.60   % m  injectors and valve feed system
p3= 0.101325*10^6      %Atmospheric pressure

% CALCULATIONS %

%mP= m_p2*(exp(D_v/(Isp2*g))-1)/(1.07-0.07*exp(D_v/(Isp2*g)));  %propellant
mass
```

```
m_stage=25121.1                %total mass
m_st=m_stage*(0.2667)      %strucure mass
mP=m_stage-m_st      % PRTOTYPE MASS propellant STAGE 1

m_T2=mP+m_st+m_p2

mdot=F_v/(Isp2*g)        % mass flow rate

mdotf=mdot/(r+1)         % mass flow rate fuel
mdot0=mdot*r/(r+1)       % mass flow gas

mdotfTC=mdotf*0.98       % mass flow rate fuel in thrust chamber
mdot0TC=mdot0*0.98       % mass flow rate gas in thrust chamber
mdotfgg= 0.02*mdotf;
mdot0gg= 0.02*mdot0;


mdotp= mdot0TC-  mdotfTC    %    mass flow rate propellant

tb=mP/mdot          %  Duration

   % Thrust coefficioent
n=((k+1)/(2*(k-1)))
M2=vpasolve(ee==(1/m)*((2+(k-1)*(m^2))/(k+1))^n ,m)

p2=p1/(1+((k-1)/2)*M2^2)^(k/(k-1))
pt=p1*((2/(k+1))^(k/(k-1)))

C_F=((((2*k^2)/(k-1))*((2/(k+1))^((k+1)/(k-1)))*((1-(p2/p1)^((k-
1)/k))))^(1/2))+((p2-p3)/p1)*ee
Isp222=(C_F*c_star)/g

At=F_v/(C_F*p1)       % Area throat
A2=ee*At              % Area exit area nozzle
T2= (k/R)*(((p2*A2*M2)/mdot)^2)
T1=T2*(1+((k-1)/2)*M2^2)
Tt=(2*T1)/(k+1)

dt=sqrt(4*At/pi)       % Diameter at throat
d2=dt*sqrt(ee)        % Diameter exit



r_v=r*(df/do)
vt=mP/da               %Total volume

vtp=vt/(r_v+1)       % volume  fuel
vto=(vt*r_v)/(r_v+1)   % volume  oxydizer

Ltp= (vtp)/(pi*((Dtank/2)^2))
Lto=(vto)/(pi*((Dtank/2)^2))

Ltpp=Ltp*1.06      %Lenght tank fuel
Ltop=Lto*1.06      % Lenght tank oxydizer
```

```
L2=0.8*((d2-dt)/(2*tand(15)))      % Chamber temperature

L_c=0.4259 %m From dt and Previous chamber test
dc=2*dt
Ac=pi*(dc/2)^2
VC= L_c*Ac      %area of the chamber

%L_c=1.1*VC2/Ac       % Lenght of the chamber
L_c2=1.1*VC/Ac

LT=L2+Ltop+Ltpp+L_c2+Li

%t111=vpasolve(VC2==mdot*ts*R*TT/p1)
%T22= (k/R)*((p2*A2*M2)/mdot)^2

fprintf('Mass propellant= %.2f \n',mP)
fprintf('Burning time = %f\n',tb)
fprintf('Total mass flow = %f\n',mdot) ;
fprintf('Mass flow Oxidizer in gas generator = %f\n',mdot0gg)
fprintf('Mass flow Fuel in gas generator= %f\n', mdotfgg)
fprintf('Mass flow Oxidizer in thrust chamber = %f\n' , mdot0TC)
fprintf('Mass flow Fuel in thrust chamber=%f\n' , mdotfTC)
fprintf('Exit Mach =%f \n' , M2)
fprintf('Exit pressure =%f \n' , p2)
fprintf('Throat pressure =%f\n' , pt)
fprintf('Chamber temperature =%f\n', T1)
fprintf('Throat temperature =%f\n',Tt)
fprintf('Exit temperature =%f\n',T2)
fprintf('Throat Area=%f\n' , At)
fprintf('Throat dimater =%f\n', dt)
fprintf('Nozzle exit area =%f\n', A2)
fprintf('Nozzle exit diameter=%f\n',d2)
fprintf('Nozzle exit lenght=%f\n' , L2)
fprintf('Thrust chamber volume=%f\n', VC )
fprintf('Thrust chamber area=%f\n' ,Ac)
fprintf('Thrust chamber diameter=%f\n',dc)
fprintf('Thrust chamber lenght=%f\n' ,L_c2)
fprintf('Oxidizer tank volume =%f\n' , vto)
fprintf('Oxidizer tank diamater=%f\n',Dtank)
fprintf('Fuel tank volume =%f\n',vtp)
fprintf('Fuel tank diamater =%f\n', Dtank)
fprintf('Oxidizer tank lenght =%f\n', Ltpp)
fprintf('Fuel tank lenght =%f\n', Ltop)
fprintf('TOTAL LENGHT=%f\n',LT)
fprintf('TOTAL MASS STAGE 1 =%f\n',m_stage)
fprintf('TOTAL MASS =%f\n',m_T2)
fprintf('STRUCTURE MASS STAGE 1 =%f\n', m_st)
fprintf('PROPELLANT MASS STAGE 1 =%f\n', mP)
```

OptiProtoStage2Tanks CODE

```
clear all
```

```
clc
%STAGE 2 PROTOTYPE WITH 500 KG
%   CHAMBER DESIGN PARAMETERS %
syms w m real;
m_p= 150     ;% Kg     payload mass
D_v= 5300      ;%    m/s
F_v= 22000   ;%    Newtons   from electron stage 2
Isp2=345  ; % s @ vaccum
g=9.806  ;   %m/s^2
r= 2.3
c_star= 1820    % cstar characteristic velocity table 5-5 RP1
p1=  10.8*10^6  %Pa    chamber pressu^re manual Rocket
df=806          %   Density fuel
do=1141          %   Density oxidizer
ee= 16      % selected nozzle area ratio      AREA RATIO FROM AUSTRONAUTIX
da= 1014     % average density
k=1.23

R=8314.3/23% Gas constant
Dtank=1.1    % assumption of 1.2 meter for design in first stage and previous
rocket
ts=1*10^-3
Li= 0.30   % m  injectors and valve feed system

% CALCULATIONS %

m_stage=3482.96               %total mass
m_st=m_stage*(0.12)     %strucure mass
mP=m_stage-m_st     % PRTOTYPE MASS propellant STAGE 1

m_T2=mP+m_st+m_p

mdot=F_v/(Isp2*g)       % mass flow rate

mdotf=mdot/(r+1)        % mass flow rate fuel
mdot0=mdot*r/(r+1)      % mass flow gas

mdotfTC=mdotf*0.98      % mass flow rate fuel in thrust chamber
mdot0TC=mdot0*0.98      % mass flow rate gas in thrust chamber
mdotfgg= 0.02*mdotf;
mdot0gg= 0.02*mdot0;

mdotp= mdot0TC-  mdotfTC    %    mass flow rate propellant

tb=mP/mdot          %  Duration

C_F=Isp2*g/c_star    % Thrust coefficioent

At=F_v/(C_F*p1)       % Area throat
A2=ee*At              % Area exit area nozzle

dt=sqrt(4*At/pi)       % Diameter at throat
d2=dt*sqrt(ee)        % Diameter exit

n=((k+1)/(2*(k-1)))
M2=vpasolve(ee==(1/m)*((2+(k-1)*(m^2))/(k+1))^n ,m)
```

```
r_v=r*(df/do)
vt=mP/da               %Total volume

vtp=vt/(r_v+1)       % volume  fuel
vto=(vt*r_v)/(r_v+1)   % volume  oxydizer

Ltp= (vtp)/(pi*((Dtank/2)^2))
Lto=(vto)/(pi*((Dtank/2)^2))

Ltpp=Ltp*1.06      %Lenght tank fuel
Ltop=Lto*1.06      % Lenght tank oxydizer


p2=p1/(1+((k-1)/2)*M2^2)^(k/(k-1))
T2= (k/R)*(((p2*A2*M2)/mdot)^2)

T1=T2*(1+((k-1)/2)*M2^2)

pt=p1*((2/(k+1))^(k/(k-1)))
Tt=(2*T1)/(k+1)

L2=0.8*((d2-dt)/(2*tand(15)))     %  nozzle lenght

L_c=0.4259 %  From dt and Previous chamber test
dc=2*dt
Ac=pi*(dc/2)^2
VC= L_c*Ac      %area of the chamber

%L_c=1.1*VC2/Ac       % Lenght of the chamber
L_c2=1.1*VC/Ac

LT=L2+Ltop+Ltpp+L_c2+Li

%t111=vpasolve(VC2==mdot*ts*R*TT/p1)
%T22= (k/R)*((p2*A2*M2)/mdot)^2

fprintf('Mass propellant= %.2f \n',mP)
fprintf('Burning time = %f\n',tb)
fprintf('Total mass flow = %f\n',mdot) ;
fprintf('Mass flow Oxidizer in gas generator = %f\n',mdot0gg)
fprintf('Mass flow Fuel in gas generator= %f\n', mdotfgg)
fprintf('Mass flow Oxidizer in thrust chamber = %f\n' , mdot0TC)
fprintf('Mass flow Fuel in thrust chamber=%f\n' , mdotfTC)
fprintf('Exit Mach =%f \n' , M2)
fprintf('Exit pressure =%f \n' , p2)
fprintf('Throat pressure =%f\n' , pt)
fprintf('Chamber temperature =%f\n', T1)
fprintf('Throat temperature =%f\n',Tt)
fprintf('Exit temperature =%f\n',T2)
fprintf('Throat Area=%f\n' , At)
fprintf('Throat dimater =%f\n', dt)
fprintf('Nozzle exit area =%f\n', A2)
fprintf('Nozzle exit diameter=%f\n',d2)
fprintf('Nozzle exit lenght=%f\n' , L2)
fprintf('Thrust chamber volume=%f\n', VC )
fprintf('Thrust chamber area=%f\n' ,Ac)
fprintf('Thrust chamber diameter=%f\n',dc)
```

```
fprintf('Thrust chamber lenght=%f\n' ,L_c2)
fprintf('Oxidizer tank volume =%f\n' , vto)
fprintf('Oxidizer tank diameter=%f\n',Dtank)
fprintf('Fuel tank volume =%f\n',vtp)
fprintf('Fuel tank diameter =%f\n', Dtank)
fprintf('Oxidizer tank lenght =%f\n', Ltpp)
fprintf('Fuel tank lenght =%f\n', Ltop)
fprintf('TOTAL LENGHT=%f\n',LT)
fprintf('TOTAL MASS STAGE 2 =%f\n',m_stage)
fprintf('TOTAL MASS =%f\n',m_T2)
fprintf('STRUCTURE MASS STAGE 1 =%f\n', m_st)
fprintf('PROPELLANT MASS STAGE 1 =%f\n', mP)
```

Solve5D

```
% Model to calculate optimal stages for a given mission.
%==========================================================================
% Run with function "Opt_stg" //  20200729 - by Elcio
%==========================================================================
clear all
clc
global Rv sigma beta

Mpl = 150;
Isp1= 311; Isp2= 345; Dv= 9400;
Rv = Dv/(9.80665*Isp1);
sigma = [0.1; 0.1];   % input to run the code sigma percentage of masses
beta = [Isp1/Isp1; Isp2/Isp1];

fun = @Opt_stg;   %
x0 = [0.1,0.1,0.1];   % initial guess of sigma
x = fsolve(fun,x0);

lT = x(1)*x(2);
m02 = Mpl / x(2);   % second stage and payload
m01 = m02 / x(1);    % first stage mass  and payload mass for first stage

m2 = m02-Mpl;    % structure plus propellant mass minus payload
m1 = m01-m02;    % fist stage mass
mtot = m01;

Y1 = ['lbda_1: ', num2str(x(1)), ';  lbda_2: ', num2str(x(2))];
Y2 = ['p: ', num2str(x(3))];
Y3 = ['lbda_T: ', num2str(lT)];
Y4 = ['m_01: ', num2str(m01), ';  m_02: ', num2str(m02)];
Y5 = ['m_stg1: ', num2str(m1), ';  m_stg2: ', num2str(m2), ';  m_PL: ',
num2str(Mpl)];

disp(Y1)
disp(Y2)
disp(Y3)
disp(Y4)
disp(Y5)
```

Opt_stg

```
function F = Opt_stg(x)
% x(1)=lbda_1; x(2)=lbda_2; x(3)=p;

global Rv sigma beta

%================================================================
F(1) = 1/x(1)+ x(3)*beta(1)*((1-sigma(1))/(sigma(1)+(1-sigma(1))*x(1)));

F(2) = 1/x(2) + x(3)*beta(2)*((1-sigma(2))/(sigma(2)+(1-sigma(2))*x(2)));

F(3) = Rv + beta(1)* log(sigma(1) + (1-sigma(1))*x(1)) + ...
    beta(2)* log(sigma(2) + (1-sigma(2))*x(2));
%================================================================
```

Example_Falcon.m

```
% wwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwww
function Example_Falcon
% wwwwwwwwwwwwwwwwwwwwwwwwwwwwwww
%{
This program numerically integrates Equations 11.6 through
11.8 for a gravity turn trajectory.
User M-functions required: rkf45
User subfunction required: rates
%}
% ------------------------
clear all;
close all;
clc

deg =pi/180; % ...Convert degrees to radians
g0 =9.81; % ...Sea-level acceleration of gravity (m/s)
Re = 6378e3; % ...Radius of the earth (m)
hscale = 7.5e3; % ...Density scale height (m)
rho0 =1.225; % ...Sea level density of atmosphere (kg/m^3)
diam = 3.6 % ...Vehicle diameter (m)
A =pi/4*(diam)^2; % ...Frontal area (m^2)
CD = 0.5; % ...Drag coefficient (assumed constant)
m0 = 542762; % ...Lift-off mass (kg)
n = 3.14; % ...Mass ratio stage 1
T2W = 1.4; % ...Thrust to weight ratio
Isp = 311; % ...Specific impulse (s)
mfinal = m0/n; % ...Burnout mass (kg)
Thrust = 7607000; % ...Rocket thrust (N)
m_dot = Thrust/Isp/g0; % ...Propellant mass flow

mprop=m0-mfinal;

tburn = mprop/m_dot; % ...Burn time (s)
hturn = 130; % ...Height at which pitchover begins (m)
```

```
t0 = 0; % ...Initial time for the numerical integration
tf = tburn; % ...Final time for the numerical integration
tspan = [t0,tf]; % ...Range of integration
% ...Initial conditions:
v0 = 0; % ...Initial velocity (m/s)
gamma0 = 89.85*deg; % ...Initial flight path angle (rad)
x0 = 0; % ...Initial downrange distance (km)
h0 = 0; % ...Initial altitude (km)
vD0 = 0; % ...Initial value of velocity loss due
% to drag (m/s)
vG0 = 0; % ...Initial value of velocity loss due
% to gravity (m/s)
%...Initial conditions vector:
f0 = [v0; gamma0; x0; h0; vD0; vG0];
%...Call to Runge-Kutta numerical integrator ,Äòrkf45,Äô
% rkf45 solves the system of equations df/dt = f(t):
[t,f] = rkf45(@rates, tspan, f0);
%...t is the vector of times at which the solution is evaluated
%...f is the solution vector f(t)
%...rates is the embedded function containing the df/dt,Äôs
% ...Solution f(t) returned on the time interval [t0 tf]:
v = f(:,1)*1.e-3; % ...Velocity (km/s)
gamma = f(:,2)/deg; % ...Flight path angle (degrees)
x = f(:,3)*1.e-3; % ...Downrange distance (km)
h = f(:,4)*1.e-3; % ...Altitude (km)
vD = -f(:,5)*1.e-3; % ...Velocity loss due to drag (km/s)
vG = -f(:,6)*1.e-3; % ...Velocity loss due to gravity (km/s)
for i = 1:length(t)
Rho = rho0 * exp(-h(i)*1000/hscale); %...Air density
q(i) = 1/2*Rho*v(i)^2; %...Dynamic pressure
end
output
return
%wwwwwwwwwwwwwwwwwwwwwwwwww
function dydt = rates(t,y)
%wwwwwwwwwwwwwwwwwwwwwwwwww

% Calculates the time rates df/dt of the variables f(t)
% in the equations of motion of a gravity turn trajectory.
%-------------------------
%...Initialize dfdt as a column vector:
dfdt = zeros(6,1);
v = y(1); % ...Velocity
gamma = y(2); % ...Flight path angle
x = y(3); % ...Downrange distance
h = y(4); % ...Altitude
vD = y(5); % ...Velocity loss due to drag
vG = y(6); % ...Velocity loss due to gravity
%...When time t exceeds the burn time, set the thrust
% and the mass flow rate equal to zero:
if t < tburn
m = m0 - m_dot*t; % ...Current vehicle mass
T = Thrust; % ...Current thrust
else
m = m0 - m_dot*tburn; % ...Current vehicle mass
T = 0; % ...Current thrust
end
g = g0/(1 + h/Re)^2; % ...Gravitational variation
```

```
% with altitude h
rho = rho0 * exp(-h/hscale); % ...Exponential density variation
% with altitude
D = 1/2 * rho*v^2 * A * CD; % ...Drag [Equation 11.1]
%...Define the first derivatives of v, gamma, x, h, vD and vG
% ("dot" means time derivative):
%v_dot = T/m - D/m - g*sin(gamma); % ...Equation 11.6


%...Start the gravity turn when h = hturn:
if h <= hturn
gamma_dot = 0;
v_dot = T/m - D/m - g;
x_dot = 0;
h_dot = v;
vG_dot = -g;
else
v_dot = T/m - D/m - g*sin(gamma);
gamma_dot = -1/v*(g - v^2/(Re + h))*cos(gamma);% ...Equation 11.7
x_dot = Re/(Re + h)*v*cos(gamma); % ...Equation 11.8(1)
h_dot = v*sin(gamma); % ...Equation 11.8(2)
vG_dot = -g*sin(gamma); % ...Gravity loss rate
end

vD_dot = -D/m; % ...Drag loss rate
%...Load the first derivatives of f(t) into the vector dfdt:
dydt(1) = v_dot;
dydt(2) = gamma_dot;
dydt(3) = x_dot;
dydt(4) = h_dot;
dydt(5) = vD_dot;
dydt(6) = vG_dot;
end
%wwwwwwwwwwwwwww
function output
%wwwwwwwwwwwwwww
fprintf('\n\n ----------------------------------\n')
fprintf('\n Initial flight path angle = %10g deg ',gamma0/deg)
fprintf('\n Pitchover altitude = %10g m ',hturn)
fprintf('\n Burn time = %10g s ',tburn)
fprintf('\n Final speed = %10g km/s',v(end))
fprintf('\n Final flight path angle = %10g deg ',gamma(end))
fprintf('\n Altitude = %10g km ',h(end))
fprintf('\n Downrange distance = %10g km ',x(end))
fprintf('\n Drag loss = %10g km/s',vD(end))
fprintf('\n Gravity loss = %10g km/s',vG(end))
fprintf('\n\n ----------------------------------\n')
figure(1)
plot(x, h)
axis equal
xlabel('Downrange Distance (km)')
ylabel('Altitude (km)')
axis([-inf, inf, 0, inf])
grid
figure(2)
subplot(2,1,1)
plot(h, v)
xlabel('Altitude (km)')
```

```
ylabel('Speed (km/s)')
axis([-inf, inf, -inf, inf])
grid
subplot(2,1,2)
plot(t, gamma)
xlabel('Time (s)')
ylabel('Flight path angle (deg)')
axis([-inf, inf, -inf, inf])
grid
figure(3)
plot(h, q)
xlabel('Altitude (km)')
ylabel('Dynamic pressure (N/m^2)')
axis([-inf, inf, -inf, inf])
grid
end %output
end %Example_Falcon
% wwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwwww
```

rkf45.m

```
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
function [tout, yout] = rkf45(ode_function, tspan, y0, tolerance)
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
%{
  This function uses the Runge-Kutta-Fehlberg 4(5) algorithm to
  integrate a system of first-order differential equations
  dy/dt = f(t,y).
  y              - column vector of solutions
  f              - column vector of the derivatives dy/dt
  t              - time
  a              - Fehlberg coefficients for locating the six solution
                   points (nodes) within each time interval.
  b              - Fehlberg coupling coefficients for computing the
                   derivatives at each interior point
  c4             - Fehlberg coefficients for the fourth-order solution
  c5             - Fehlberg coefficients for the fifth-order solution
  tol            - allowable truncation error
  ode_function   - handle for user M-function in which the derivatives f
                   are computed
  tspan          - the vector [t0 tf] giving the time interval for the
                   solution
  t0             - initial time
  tf             - final time
  y0             - column vector of initial values of the vector y
  tout           - column vector of times at which y was evaluated
  yout           - a matrix, each row of which contains the components of y
                   evaluated at the correponding time in tout
  h              - time step
  hmin           - minimum allowable time step
  ti             - time at the beginning of a time step
  yi             - values of y at the beginning of a time step
  t_inner        - time within a given time step
  y_inner        - values of y witin a given time step
  te             - trucation error for each y at a given time step
  te_allowed     - allowable truncation error
```

```
  te_max         - maximum absolute value of the components of te
  ymax           - maximum absolute value of the components of y
  tol            - relative tolerance
  delta          - fractional change in step size
  eps            - unit roundoff error (the smallest number for which
                   1 + eps > 1)
  eps(x)         - the smallest number such that x + eps(x) = x
  User M-function required: ode_function
%}
% ----------------------------------------------------------------

a = [0 1/4 3/8 12/13 1 1/2];

b = [    0          0          0          0         0
        1/4         0          0          0         0
        3/32       9/32        0          0         0
     1932/2197 -7200/2197  7296/2197      0         0
      439/216      -8      3680/513   -845/4104     0
       -8/27        2     -3544/2565  1859/4104  -11/40];

c4 = [25/216  0  1408/2565    2197/4104   -1/5    0  ];
c5 = [16/135  0  6656/12825  28561/56430  -9/50  2/55];

if nargin < 4
    tol  = 1.e-8;
else
    tol = tolerance;
end

t0   = tspan(1);
tf   = tspan(2);
t    = t0;
y    = y0;
tout = t;
yout = y';
h    = (tf - t0)/100; % Assumed initial time step.

while t < tf
    hmin = 16*eps(t);
    ti   = t;
    yi   = y;
    %...Evaluate the time derivative(s) at six points within the current
    %   interval:
    for i = 1:6
        t_inner = ti + a(i)*h;
        y_inner = yi;
        for j = 1:i-1
            y_inner = y_inner + h*b(i,j)*f(:,j);
        end
        f(:,i) = feval(ode_function, t_inner, y_inner);
    end

    %...Compute the maximum truncation error:
    te     = h*f*(c4' - c5'); % Difference between 4th and
                              % 5th order solutions
    te_max = max(abs(te));
```

```matlab
    %...Compute the allowable truncation error:
    ymax       = max(abs(y));
    te_allowed = tol*max(ymax,1.0);

    %...Compute the fractional change in step size:
    delta = (te_allowed/(te_max + eps))^(1/5);

    %...If the truncation error is in bounds, then update the solution:
    if te_max <= te_allowed
        h      = min(h, tf-t);
        t      = t + h;
        y      = yi + h*f*c5';
        tout   = [tout;t];
        yout   = [yout;y'];
    end

    %...Update the time step:
    h  = min(delta*h, 4*h);
    if h < hmin
        fprintf(['\n\n Warning: Step size fell below its minimum\n'...
                 ' allowable value (%g) at time %g.\n\n'], hmin, t)
        return
    end
end
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
```

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TABLE 5–5. Theoretical Chamber Performance of Liquid Rocket Propellant Combinations

		Mixture Ratio						I_s (sec)		
Oxidizer	Fuel	By Mass	By Volume	Average Specific Gravity	Chamber Temp.(K)	Chamber c^* (m/sec)	$\mathfrak{M}$, (kg/mol)	Shifting	Frozen	k
Oxygen	Methane	3.20	1.19	0.81	3526	1835	20.3		296	1.20
		3.00	1.11	0.80	3526	1853		311		
	Hydrazine	0.74	0.66	1.06	3285	1871	18.3		301	1.25
		0.90	0.80	1.07	3404	1892	19.3	313		
	Hydrogen	3.40	0.21	0.26	2959	2428	8.9		386	1.26
		4.02	0.25	0.28	2999	2432	10.0	389.5		
	RP-1	2.24	1.59	1.01	3571	1774	21.9	300	285.4	1.24
		2.56	1.82	1.02	3677	1800	23.3			
	UDMH	1.39	0.96	0.96	3542	1835	19.8		295	1.25
		1.65	1.14	0.98	3594	1864	21.3	310		
Fluorine	Hydrazine	1.83	1.22	1.29	4553	2128	18.5	334		1.33
		2.30	1.54	1.31	4713	2208	19.4		365	
	Hydrogen	4.54	0.21	0.33	3080	2534	8.9		389	1.33
		7.60	0.35	0.45	3900	2549	11.8	410		
Nitrogen tetroxide	Hydrazine	1.08	0.75	1.20	3258	1765	19.5		283	1.26
		1.34	0.93	1.22	3152	1782	20.9	292		
	50% UDMH	1.62	1.01	1.18	3242	1652	21.0		278	1.24
	50% hydrazine	2.00	1.24	1.21	3372	1711	22.6	289		
	RP-1	3.4	1.05	1.23	3290		24.1		297	1.23
	MMH	2.15	1.30	1.20	3396	1747	22.3	289		
		1.65	1.00	1.16	3200	1591	21.7		278	1.23
Red fuming nitric acid	RP-1	4.1	2.12	1.35	3175	1594	24.6		258	1.22
		4.8	2.48	1.33	3230	1609	25.8	269		
	50% UDMH	1.73	1.00	1.23	2997	1682	20.6		272	1.22
	50% hydrazine	2.20	1.26	1.27	3172	1701	22.4	279		
Hydrogen peroxide (90%)	RP-1	7.0	4.01	1.29	2760		21.7		297	1.19

Notes:
Combustion chamber pressure—1000 psia (6895 kN/m^2); nozzle exit pressure—14.7 psia (1 atm); optimum expansion.
Adiabatic combustion and isentropic expansion of ideal gases.
The specific gravity at the boiling point has been used for those oxidizers or fuels that boil below 20 °C at 1 atm pressure, see Eq. 7–1.
Mixture ratios are for approximate maximum values of I_s.

Data estimating the L* for the chamber length

TABLE 11–3. Data on Three Russian Large Liquid Propellant Rocket Engines Using a Staged Combustion Cycle

Engine Designation	RD-120	RD-170	RD-253
Application (number of engines)	Zenit second stage (1)	Energia launch vehicle booster (4), Zenit first stage (1), and Atlas V (1)	Proton vehicle booster (1)
Oxidizer	Liquid oxygen	Liquid oxygen	N_2O_4
Fuel	Kerosene	Kerosene	UDMH
Number and types of turbopumps (TPs)	One main TP and Two boost TPs	One main TP and Two boost TPs	Single TP
Thrust control, %	Yes	Yes	±5
Mixture ratio control, %	±10	±7	±12
Throttling (full flow is 100%), %	85	40	None
Engine thrust (vacuum), kg	85,000	806,000	167,000
Engine thrust (SL), kg	—	740,000	150,000
Specific impulse (vacuum), sec	350	337	316
Specific impulse (SL), sec	—	309	285
Propellant flow, kg/sec	242.9	2393	528
Mixture ratio, O/F	2.6	2.63	2.67
Length, mm	3872	4000	2720
Diameter, mm	1954	3780	1500
Dry engine mass, kg	1125	9500	1080
Wet engine mass, kg	1285	10,500	1260
		Thrust Chamber Characteristics	
Chamber diameter, mm	320	380	430
Characteristic chamber length, mm	1274	1079.6	999.7
Chamber area contraction ratio	1.74	1.61	1.54
Nozzle throat diameter, mm	183.5	235.5	279.7
Nozzle exit diameter, mm	1895	1430	1431
Nozzle area ratio	106.7	36.9	26.2
Thrust chamber length, mm	2992	2261	2235
Nominal combustion temperature, K	3670	3676	3010
Rated chamber pressure, kg/cm^2	166	250	150
Nozzle exit pressure, kg/cm^2	0.13	0.73	0.7
Thrust coefficient, vacuum	1.95	1.86	1.83
Thrust coefficient, SL	—	1.71	1.65
Gimbal angle, degree	Fixed	8	Fixed
Injector type	Hot, oxidizer-rich precombustor gas plus fuel		

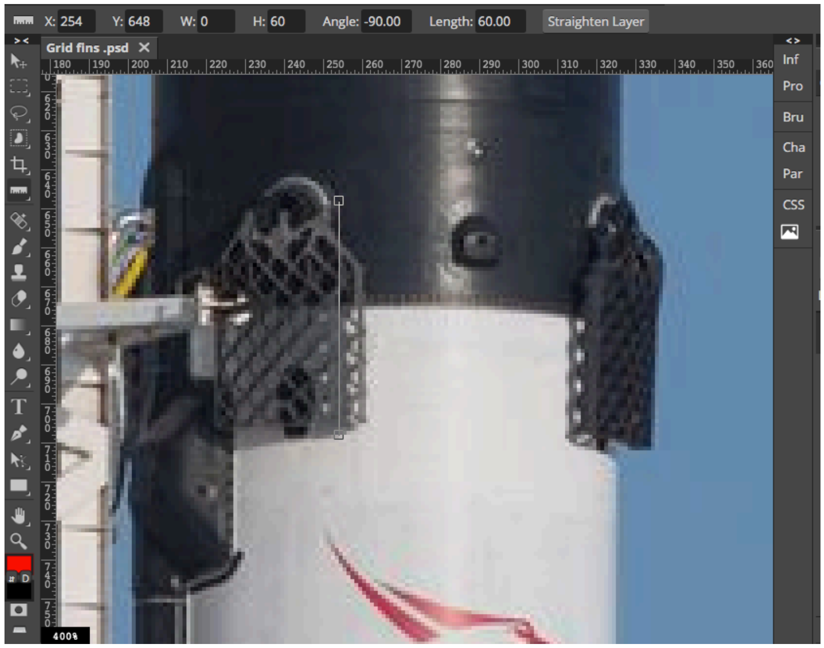
With a staged combustion cycle the thrust, propellant flow, and mixture ratio for the thrust chamber have the same values as for the entire engine.

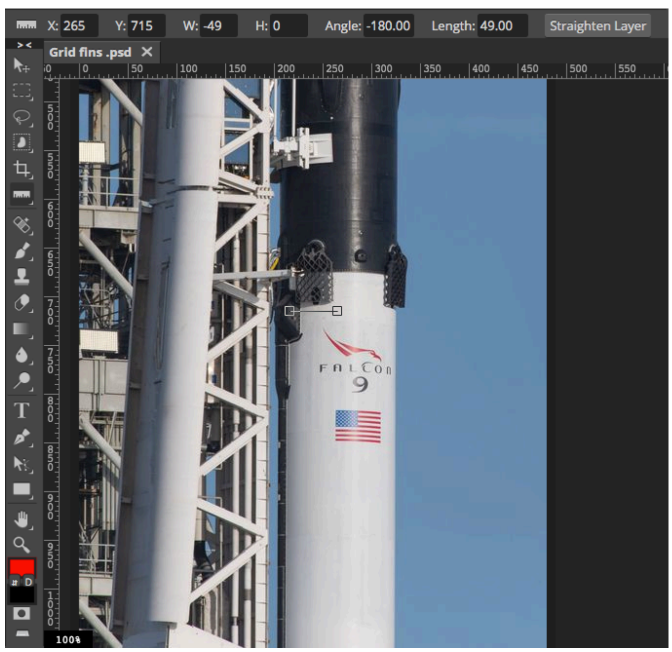
TABLE 11–3. (*Continued*)

Engine Designation	RD-120		RD-170		RD-253	
	Turbopump Characteristics[b]					
Pumped liquid	*Oxidizer*	*Fuel*	*Oxidizer*	*Fuel*	*Oxidizer*	*Fuel*
Pump discharge pressure, kg/cm^2	347	358	614	516	282	251
Flow rate, kg/sec	173	73	1792	732	384	144
Impeller diameter, mm	216	235	409	405	229	288
Number of stages	1	1	1	1 + 1[a]	1	1 + 1
Pump efficiency, %	66	65	74	74	68	69
Pump shaft power, hp	11,210	6145	175,600	77,760	16,150	8850
Required pump NPSH, m	37	23	260	118	45	38
Shaft speed, rpm	19,230		13,850		13,855	
Pump impeller type	Radial flow		Radial flow		Radial flow	
Turbine power, hp	17,588		257,360		25,490	
Turbine inlet pressure, main turbine, kg/cm^2	324		519		239	
Pressure ratio	1.76		1.94		1.42	
Turbine inlet temperature, K	735		772		783	
Turbine efficiency, %	72		79		74	
Number of turbine stages	1		1		1	
	Preburner Characteristics					
Flow rate, kg/sec	177		836		403.5	
Mixture ratio, O/F	53.8		54.3		21.5	
Chamber pressure, kg/cm^2	325		546		243	
Number of preburners	1		2		1	

[a]Fuel flow to precombustor goes through a small second-stage pump.
[b]Includes booster pump performance where applicable.
(From NPO Energomash, Khimki, Russia.)

Pictures for Fin measurements





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